

Based on our grammar, the expression  $10 - 4 + 3$

3

can be parsed either as

$(10 - 4) + 3$ ,

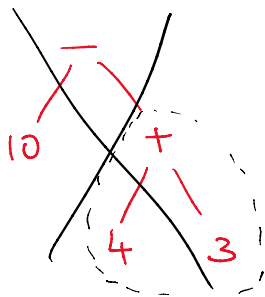
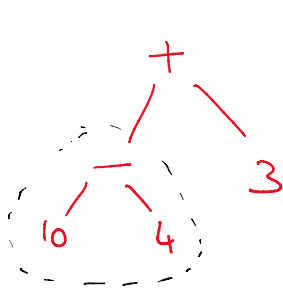
or as

10  $(- (4 + 3))$

Both these parentheses are implicit.

What if we say that the right side of a MINUS must be a single number?

$10 - 4 + 3$



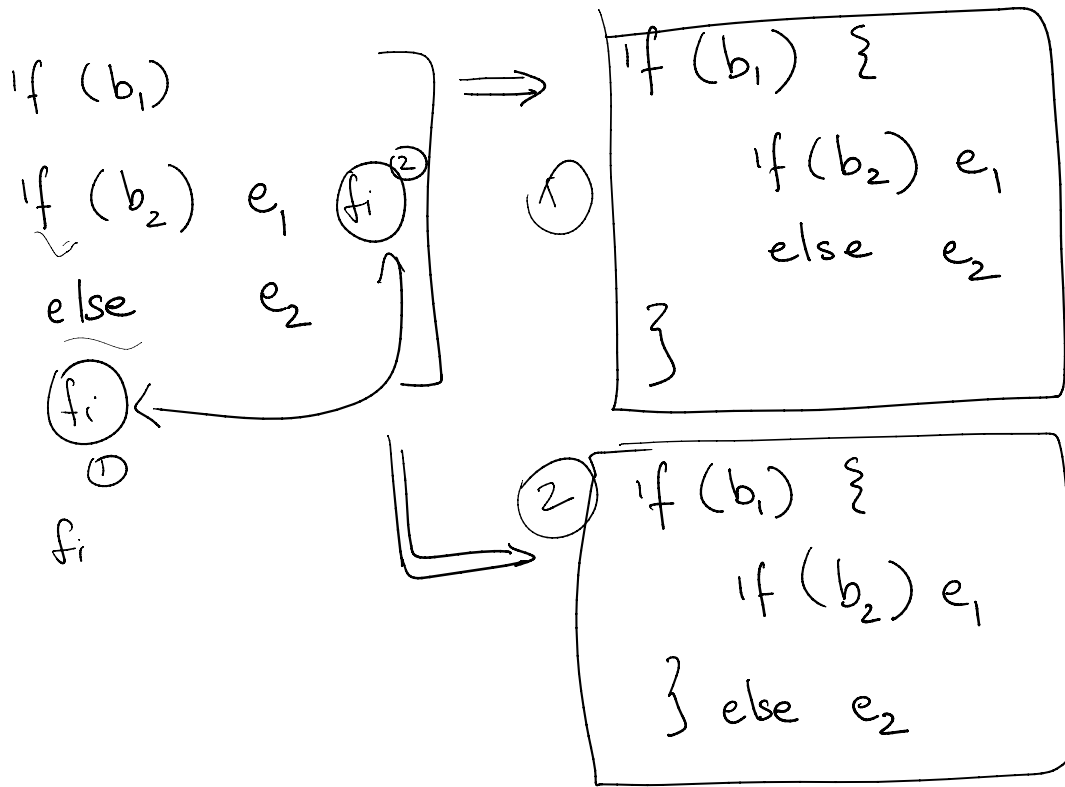
Then, this parse tree is no longer feasible.

- But what about  $10 - (4 + 3)$

$$\text{expr} ::= \dots$$

$$\left[ \begin{array}{l} | \text{expr} - \text{Int Literal} \\ | \overline{\text{expr}} = (\overline{\text{expr}}) \\ | \dots \end{array} \right]$$

if  $b_1$  then if  $b_2$  then  $e_1$  else  $e_2$



stmt ::= ...

| if (expr) then stmt fi

| if (expr) then stmt else stmt fi

How Do They Work?

# Regular expressions

" ( "	$r_1 + r_2$	$r_1 \cdot r_2$	$r^*$
concrete string	regular expr $r_1$ or regular expr $r_2$	$r_1$ followed by $r_2$	<u>Zero or more</u> occurrences of $r$ .

$$('0' - '9') = '0' + '1' + '2' + \dots + '9'$$

A string matches  $r^+$  if it can be split into one or more occurrences of  $r$ .

$$r^+ = \underbrace{r^* + r}_{\text{zero or one more}} = r^* \quad \text{X}$$

$$r^+ = r^* \cdot r \quad \checkmark$$

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Q1: Does "abc" match  $(\underbrace{'a' + 'b' + 'c'})^*$  ?

A string matches  $(\underbrace{'a' + 'b' + 'c'})^*$  if it can be broken into zero or more pieces, each of which match  $\underbrace{'a' + 'b' + 'c'}$ .

"abc" =  $\underbrace{"a"} \cdot \underbrace{"b"} \cdot \underbrace{"c"}$  . Yes.

Q2: Does "abcd" match  $(a+b+c)^*$  ?  
No.   
!!

Q3: Does "abc" match  $(a+b+c)^* \cdot c^*$  ?

$$\text{"abc"} = \underbrace{\text{"ab"}}_{(a+b+c)^*} \cdot \text{"c"}_{c^*} = \underbrace{\text{"abc"}}_{(a+b+c)^*} \cdot \underbrace{\text{" "}}_{c^*}$$



Q4: Why does "abc" match  $(a+b+c)^*$  ?

Shouldn't  $(a+b+c)^*$  mean that the same letter (a or b or c) is repeated multiple times?

Counterpoint: This would be  $a^* + b^* + c^*$

$$(a+b+c)^* \neq a^* + b^* + c^*$$

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Q5: How does one intuitively understand

$$(a+b+c)^* \cdot c^*$$

$(a+b+c)^* \cdot c^+$  ?

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$(a+b+c)^*$        $c^+$

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Q6: For example, in C,  $x5$  is a valid variable name,  $_x5$

On the other hand,  $5x$  is not a valid variable name.

alphabet + number + underscore

$(\text{alpha} + \text{underscore}) \cdot (a + n + u)^*$

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Q7: What is a regular expression for email addresses?

person @ example . com

$(A + N + D + U) \cdot @ \cdot (A + N + D + U)$   
(alpha number dot underscore)

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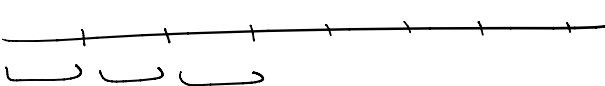
Q8: Given a string  $s$  & a regular expression  $r$ ,  
- find if  $s$  matches  $r$ .

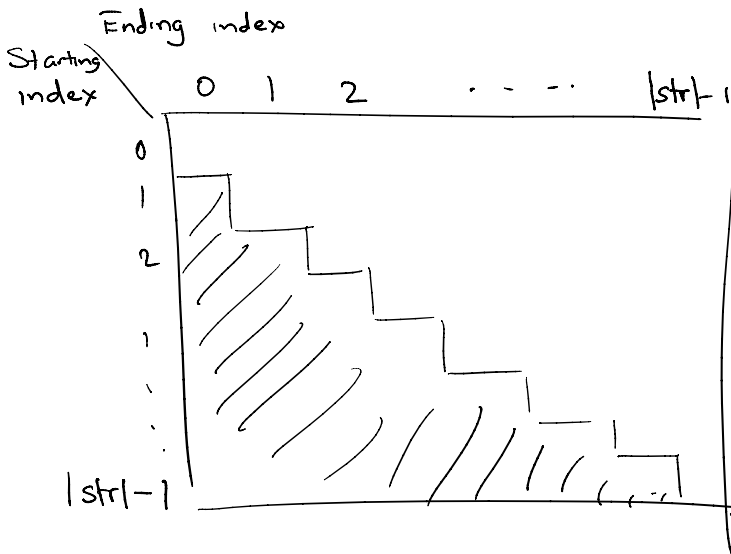
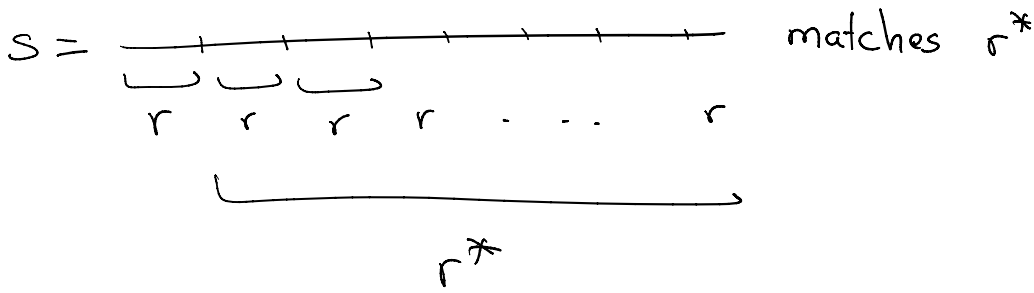
" ( " concrete string	$r_1 + r_2$ regular expr $r_1$ or regular expr $r_2$	$r_1 \cdot r_2$ $r_1$ followed by $r_2$	$r^*$ <u>Zero or more</u> occurrences of $r$ .
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The issue with greedy algorithms:

- How to know if we'll need to backtrack?

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$s =$   matches  $r^*$




regex  $r$

All subexpressions  
of  $r$

$r = \text{"str"}$

If  $|str| = n$  &  
regex  $r$  is of size  $k$   
then  $O(n^2k)$  entries  
in the table  
Subexpressions  
of  $r$ .

To fill in

table  $(i \ j \ \text{string}(s))$ ,  shape of regex

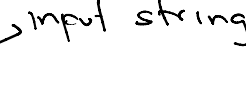
check if  $\text{str}[i..j] = s$ ,  input string

table  $(i \ j \ r_1 + r_2)$

$= \text{table}(i \ j \ r_1) \parallel \text{table}(i \ j \ r_2)$



table (i j  $r_1 \cdot r_2$ )

= Check if  $\exists k$   $i \leq k \leq j$

s.t. table (i k  $r_1$ ) & table (k j  $r_2$ )

table (i j  $r^*$ )

= Check if  $\exists k$   $i \leq k \leq j$

s.t. table (i k  $r$ ) & table (k j  $r^*$ )

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If  $|str| = n$  &  
regex  $r$  is of size  $k$   
then  $O(n^2 k)$  entries  
in the table

Filling in a single  
entry might require  
 $O(n)$  time

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On the whole, we need  $O(n^3 k)$  time  
to check if a string of size  $n$   
matches a regex of size  $k$ .