

Question: How do we express deeper forms of recursion?

Example (Non-recursive)

Cousins: Two people whose parents are siblings

Two people who share the same grandparent

Input

Parent

$c_1$	$c_2$
x	y
.	.

→ "x is a parent of y"

"x is a cousin of y"

Output

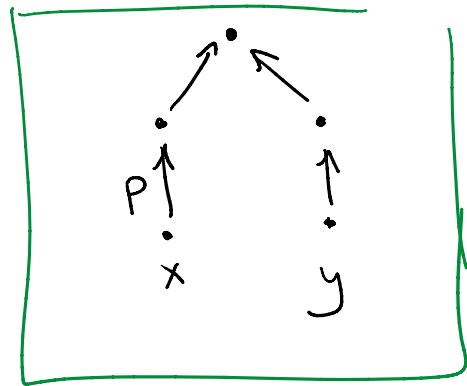
Cousin

$c_1$	$c_2$
x	y



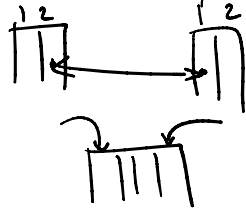
# Computing Cousins using RA / SQL

$\text{select } (P_1.c_1, P_2.c_2)$   
 from Parent as P<sub>1</sub>,  
 Parent as P<sub>2</sub>  
 where  $P_1.c_2 = P_2.c_1$



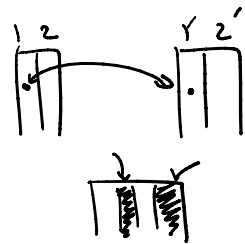
Basic graph pattern

$\pi_{1,4} (P \bowtie_{2=1'} P)$

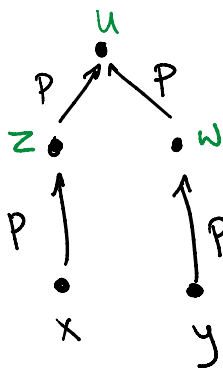


GP

$\pi_{2,4} (GP \bowtie_{1=1'} GP)$



Cousins.



For all People  $x, y, z, w, u$ ,

If we have

Parent( $x, z$ ) & Parent( $z, u$ ) &

Parent( $y, w$ ) & Parent( $w, u$ )

then we have Cousins( $x, y$ ).

) | then we have Cousins (x y).

$\forall x, y, z, w, u$   
↑  
Imphcat  
Cousins (x, y) :- P(x, z), P(z, u), P(y, w), P(w, u).

This is our first Datalog query.  
"clause" "rule".

Head  
↑  
"Output" ?

Body ← "Input" (?)

How could we write recursive Datalog queries?

Just use the same relation in  
the head & body!

$\frac{E(x, y)}{P(x, y)}$

$P(x, y) :- E(x, y).$  ← "or": Just write multiple rules  
 $P(x, z) :- E(x, y), P(y, z).$

$$P(x, z) :- E(x, y) \text{ and } P(y, z)$$

$$\frac{E(x, y) \quad P(y, z)}{P(x, z)}$$

(Note: A red dashed arrow labeled "recursion" points from the  $P(y, z)$  in the rule back to the  $P(y, z)$  in the fraction. The word "and" is written in red between the two terms in the rule.)

Theorem: Non-recursive Datalog = Relational algebra positive  
 ↑  
 In expressive power

## Recursive Datalog

What does a recursive Datalog query "mean"?

$$P(x, y) :- E(x, y) \quad \textcircled{1}$$

$$P(x, z) :- E(x, y), P(y, z) \quad \textcircled{2}$$

