

Datalog

— Becoming comfortable with the language

Writing programs

— The problem of negation

Stratified negation
(Most Datalogs)

Negation-as-failure
(Prolog)

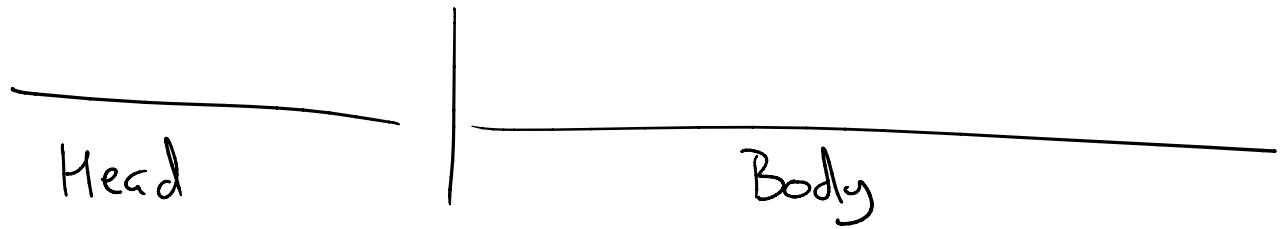
Stable model semantics
(Answer Set Programming)

↓
Aggregation

— Demystify the magic / Evaluation algorithms.

Example (Rule / Clause)

Cousin (x y) :- Parent (z_x), Parent (w_y),
Parent (u-z), Parent (u-w).

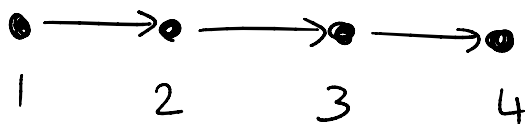


$\forall x y z w u$, If body
then head

Example

① Path (x, y) :- Edge (x, y) \rightarrow

② Path (x, z) :- Edge (x, y), Path (y, z).



Iteration 0	Iteration 1	Iteration 2	Iteration 3
E(1 2)	E(1 2)	E(1 2)	E(1 2)
E(2 3)	E(2 3)	E(2 3)	E(2 3)
E(3 4)	E(3 4)	E(3 4)	E(3 4)
	P(1 2)	P(1 2)	P(1 2)
	P(2 3)	P(2 3)	P(2 3)
	P(3 4)	P(3 4)	P(3 4)
		P(1 3)	P(1 3)
		P(2 4)	P(2 4)
			P(1, 4)

①
①
①

"Blame"
"Provenance"

— At this point, the rules

don't derive anything new

"Fixpoint" / "Fixed point"

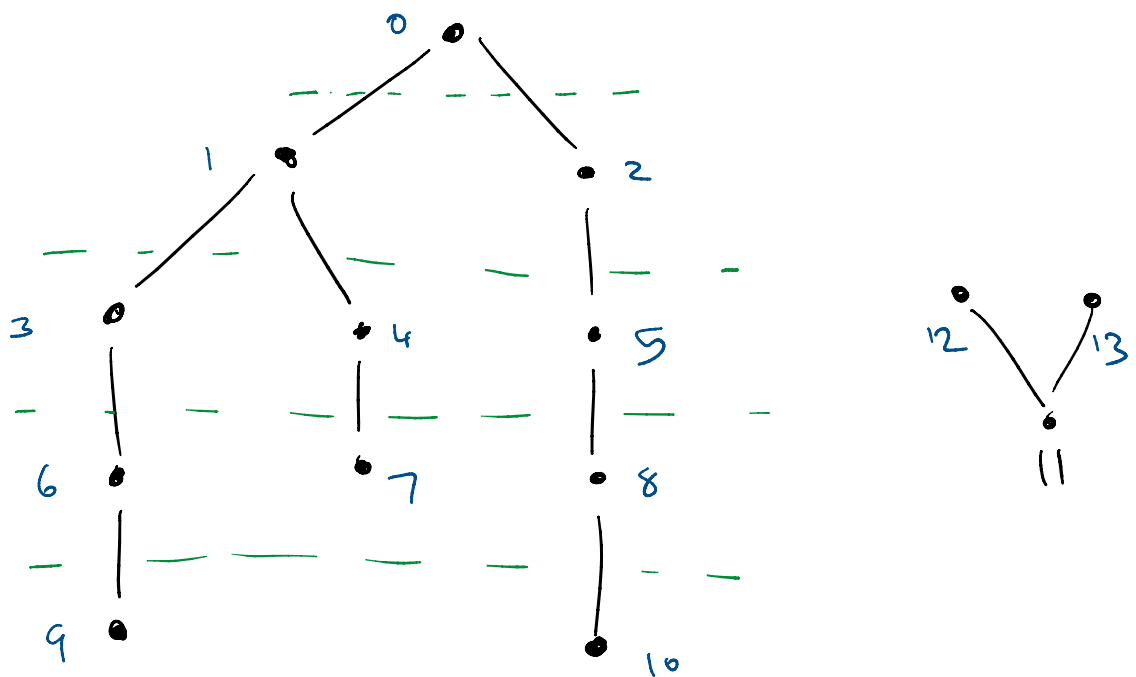
Least fixpoint

- Programs will terminate
(under some assumptions)

Example : Find all people in the same generation of a family.

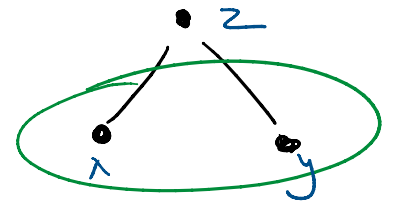
Parent (x y) — "x is a parent of y"

Samegen (x y) — "x & y are in the same generation."

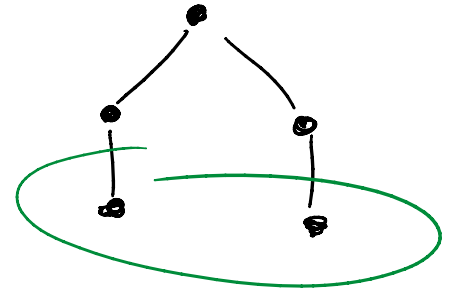


If same parent, then samegen.

$sgen(x, y) :- parent(z, x) \quad parent(z, y)$ (1)

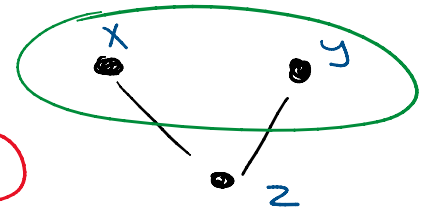


If same grandparent, then samegen

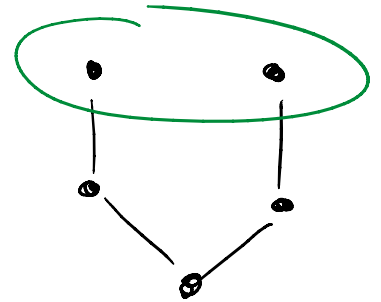


If share offspring, then samegen

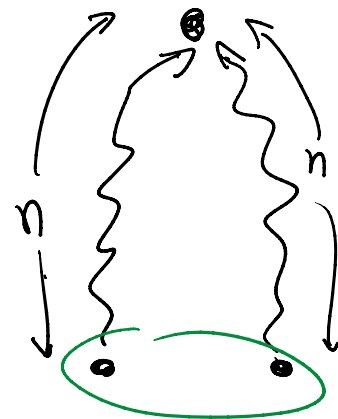
$sgen(x, y) :- parent(x, z) \quad parent(y, z)$ (2)



If share grandchild, then samegen

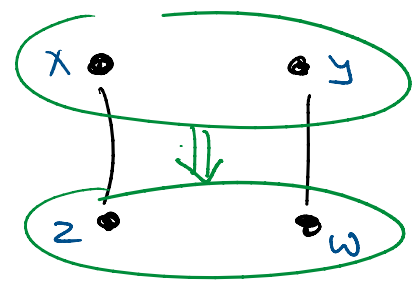


If common ancestor, &
same # of levels to common ancestor
then samegen



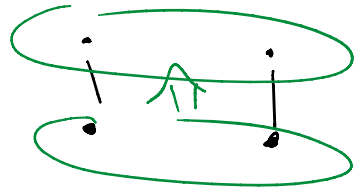
If parents in samegen,
 then offspring in samegen.
 $\text{sgen}(x\ y) :- \text{parent}(x\ z)\ \text{parent}(y\ w)\ \text{sgen}(z\ w)$

(3)

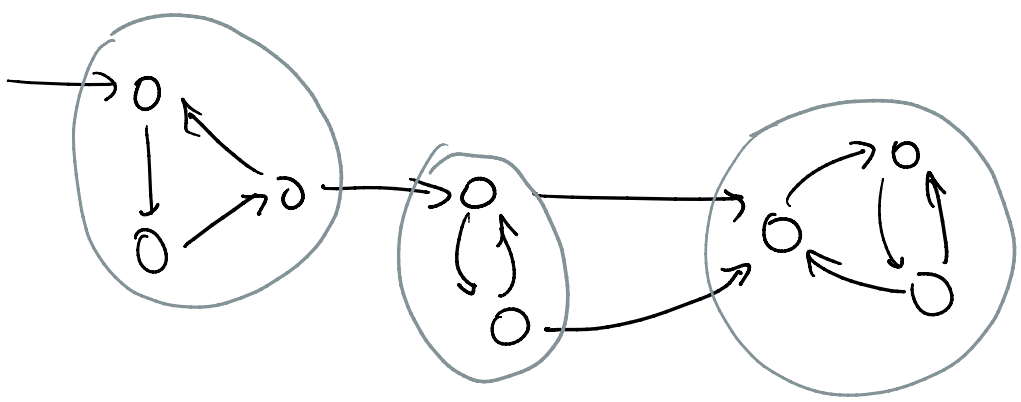


If children in samegen,
 then parents in samegen.
 $\text{sgen}(x\ y) :- \text{parent}(z\ x)\ \text{parent}(w\ y)\ \text{sgen}(w\ z)$

(4)



Example Strongly connected components



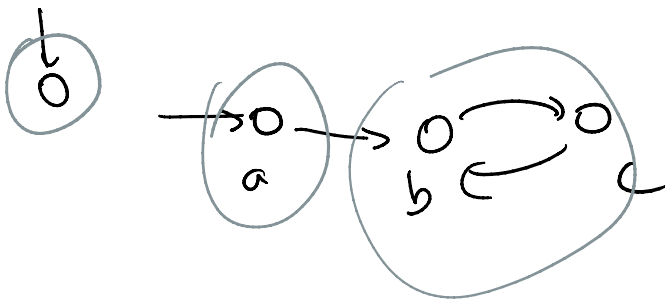
Defn: Vertices x & y are in the same SCC

if there is a path from x to y
& there is a path from y to x .

$scc(x, y) :- path(x, y) \wedge path(y, x)$

$path(x, y) :- edge(x, y)$

$path(x, z) :- path(x, y) \wedge path(y, z)$



Closed World Assumption

All that is true, I know to be true.

I know everything (that is true)

The Problem of Negation

Soln 1: If a rule has $R(_)$ in the head then $\neg R(_)$ cannot be in the body.

Paradox $(X) :- \text{rel}(X), \neg \text{Paradox}(X)$

$P_1(X) :- \text{rel}(X), \neg P_2(X)$

$P_2(X) :- P_1(X).$

Soln 1 fails to eliminate this program

Soln 2 (Stratification)

$s(e)$ \circ edge

$s(p)$ 1 path $(x, y) :- \text{edge}(x, y)$

$s(p) \geq s(e)$

path $(x, z) :- \text{edge}(x, y)$ path $(y, z).$

$s(n)$ 2 nopath $(x, y) :- \neg \text{path}(x, y)$

$s(n) \geq \text{nopath}(x, y) :- \text{! path}(x, y)$

$s(p) \geq s(e)$

$s(p) \geq s(p)$

$s(n) > s(p)$
 + strict!

$\text{nopath}(x, y) :- \text{! path}(x, y)$

But what do x & y
 range over?

$\text{nopath}(x, y) :- \text{vertex}(x) \text{ vertex}(y)$
 $\text{! path}(x, y)$
