

- Syntactic organization : Compilation units
- Namespace : C++ Namespaces, Java packages
- Abstraction : Classes, Functions
- Code reuse : Inheritance, parametric polymorphism

Vernacular	Values	Types	Functions
Module	Structure Implementation	Signature Interface	Functions

Ex 1 : Stacks

type stack = _____

push : int → stack → stack

pop : stack → int option * stack

Contract
"Signature"

```
type stack = int list
```

```
let push (a : int) (s : stack) : stack = a :: s
```

```
let pop (s : stack) : int option * stack =
```

```
  match s with
```

```
  | hd :: tl -> (Some hd, tl)
```

```
  | [] -> (None, s)
```

Impl 1

"Structure"

```
type stack2 = EmptyStack | NonEmpty of int * stack2
```

```
let push2 a s = NonEmpty(a, s)
```

```
let pop2 s =
```

```
  match s with
```

```
  | NonEmpty(a, s2) -> (Some(a), s2)
```

```
  | EmptyStack -> (None, s)
```

Impl 2

```
module type Stack =
  sig
    type stack
    val emptyStack : stack
    val push : int -> stack -> stack
    val pop : stack -> int option * stack
  end
```

```
module ListStack : Stack =
  struct
    type stack = int list
    let emptyStack = []
    let push a s = a :: s
    let pop s =
      match s with
      | [] -> (None, s)
      | hd :: tl -> (Some hd, tl)
  end
```

```
module ADTStack : Stack =
  struct
    type stack = EmptyStack | NonEmpty of int * stack
    let emptyStack = EmptyStack
    let push a s = NonEmpty(a, s)
    let pop s =
      match s with
      | EmptyStack -> (None, s)
      | NonEmpty(a, s2) -> (Some a, s2)
  end
```

Extended Stacks

```
module type ExtendedStack =  
  sig  
    type stack  
    val emptyStack : stack  
    val push : int -> stack -> stack  
    val pop : stack -> int option * stack  
    val reverse : stack -> stack  
  end
```

How to reverse a stack?

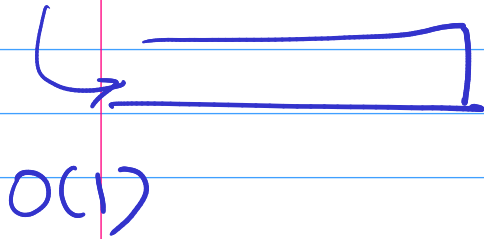
```
let reverseLS s =  
  let rec helper acc s =  
    match ListStack.pop s with  
    | (Some hd, s2) -> helper (ListStack.push hd acc) s2  
    | (None, _) -> acc in  
  helper ListStack.emptyStack s  
  
let reverseAS s =  
  let rec helper acc s =  
    match ADTStack.pop s with  
    | (Some hd, s2) -> helper (ADTStack.push hd acc) s2  
    | (None, _) -> acc in  
  helper ADTStack.emptyStack s
```

```
module Extend(S : Stack) : ExtendedStack =  
  struct  
    include S  
    let reverse s =  
      let rec helper acc s =  
        match S.pop s with  
        | (Some hd, s2) -> helper (S.push hd acc) s2  
        | (None, _) -> acc in  
      helper S.emptyStack s  
  end
```

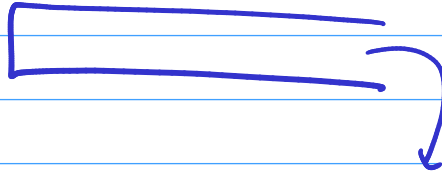
```
module ExtendedLS = Extend(ListStack)
```

```
module ExtendedAS = Extend(ADTStack)
```

Ex 2: Queue



$O(1)$



$O(n)$ worst case

$O(1)$ amortized