$$r ::= \overbrace{\varepsilon \mid a}^{\text{Base cases}} \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

Either-or   Concatenation   Kleene-*

<u>Q</u>: Given r & w, does w match r?

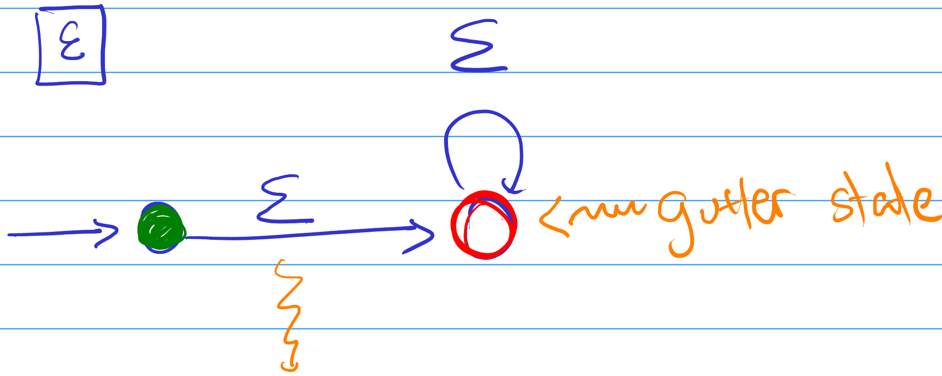
DP requires: $O(|r||w|^3)$ time

$O(|r||w|^2)$ memory.

<u>Can we do better?</u>

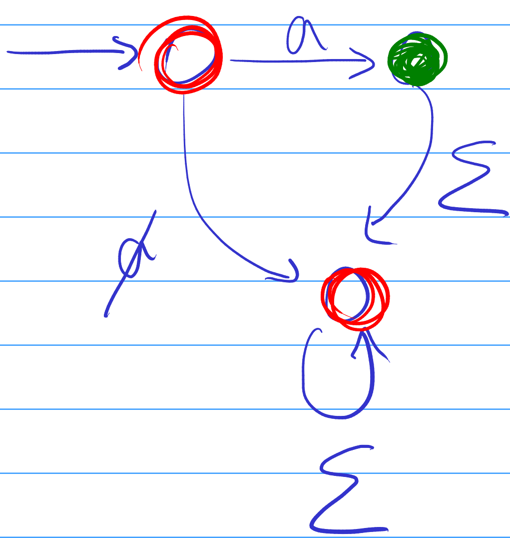
Hi*

$\boxed{\varepsilon}$

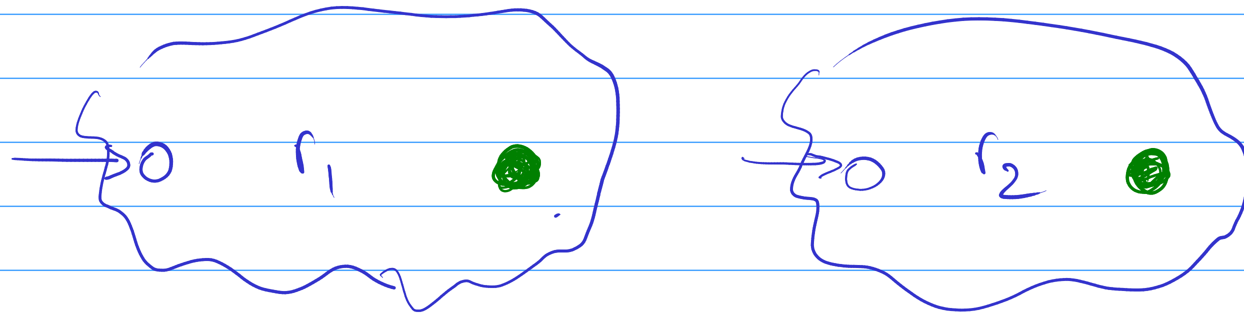


Upon seeing any character
move token

$\boxed{a}$

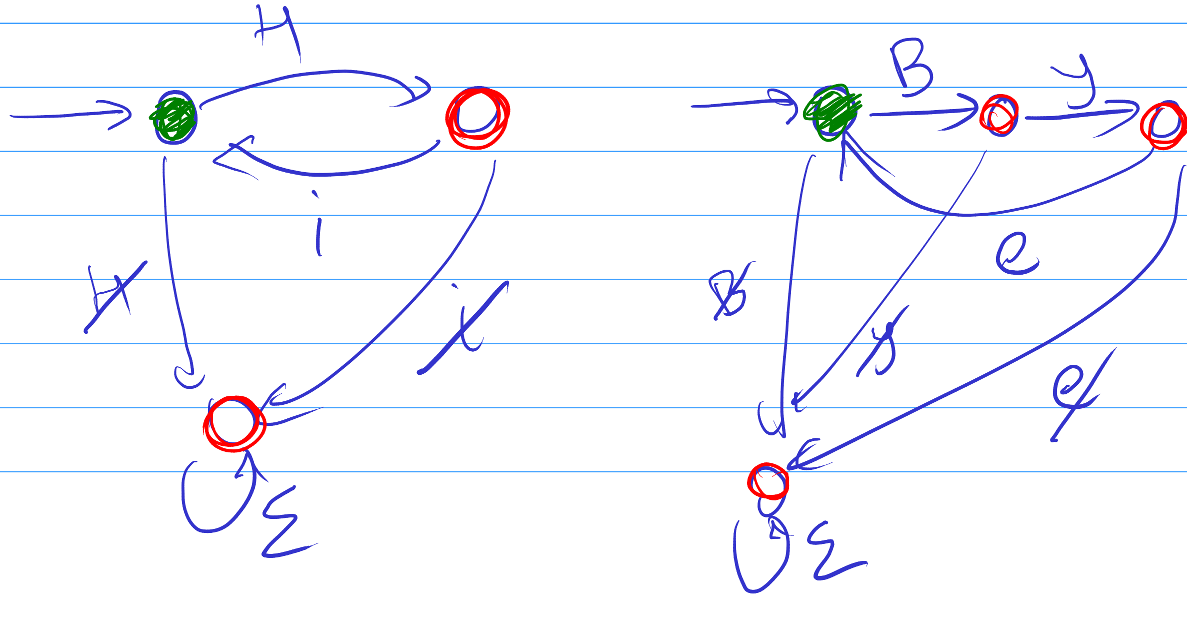


$\boxed{r_1 + r_2}$



P1 : Look at the first letters of w & decide whether to play r_1 or to play r_2.

Easy ex : $(Hi)^* + (Bye)^*$

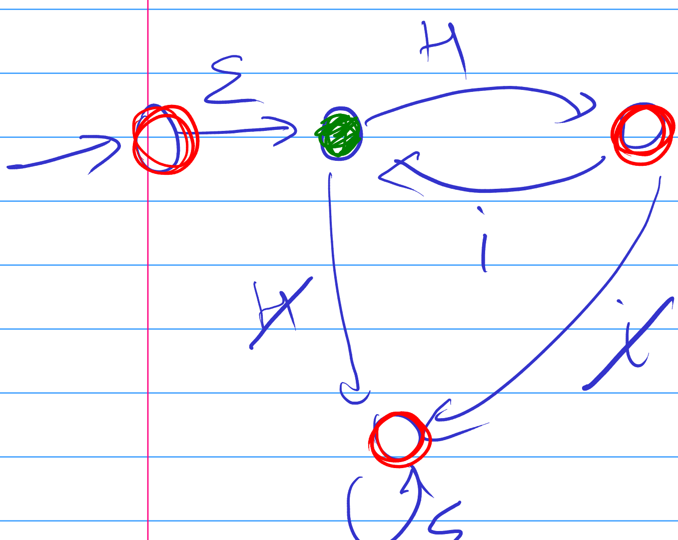


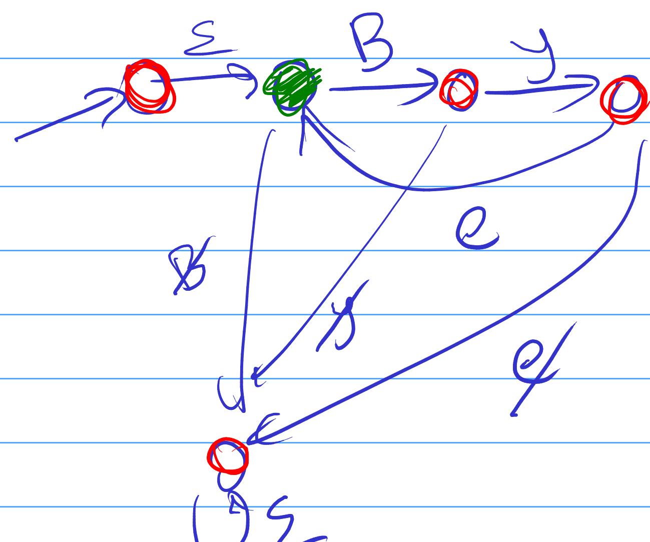
Harder ex

$$\Sigma \cdot \underbrace{(Hi)^*}\ +\ \Sigma \cdot \underbrace{(Bye)^*}$$

Don't care about the first character

The suffix following that had better have some form.



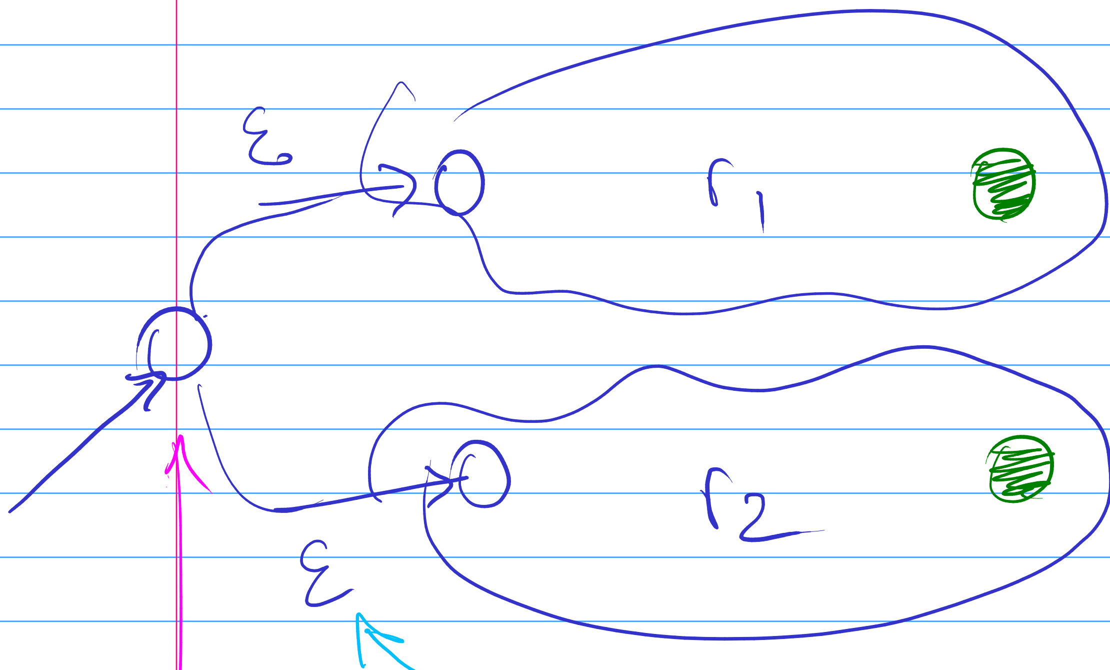


Hardest Ex

$$\overbrace{\Sigma^{A}\ \text{Hi}}^{r_1} + \overbrace{\Sigma^{*}\ \text{Bye}}^{r_2}$$

The decision of whether to play r_1 or r_2 cannot be made till the end.

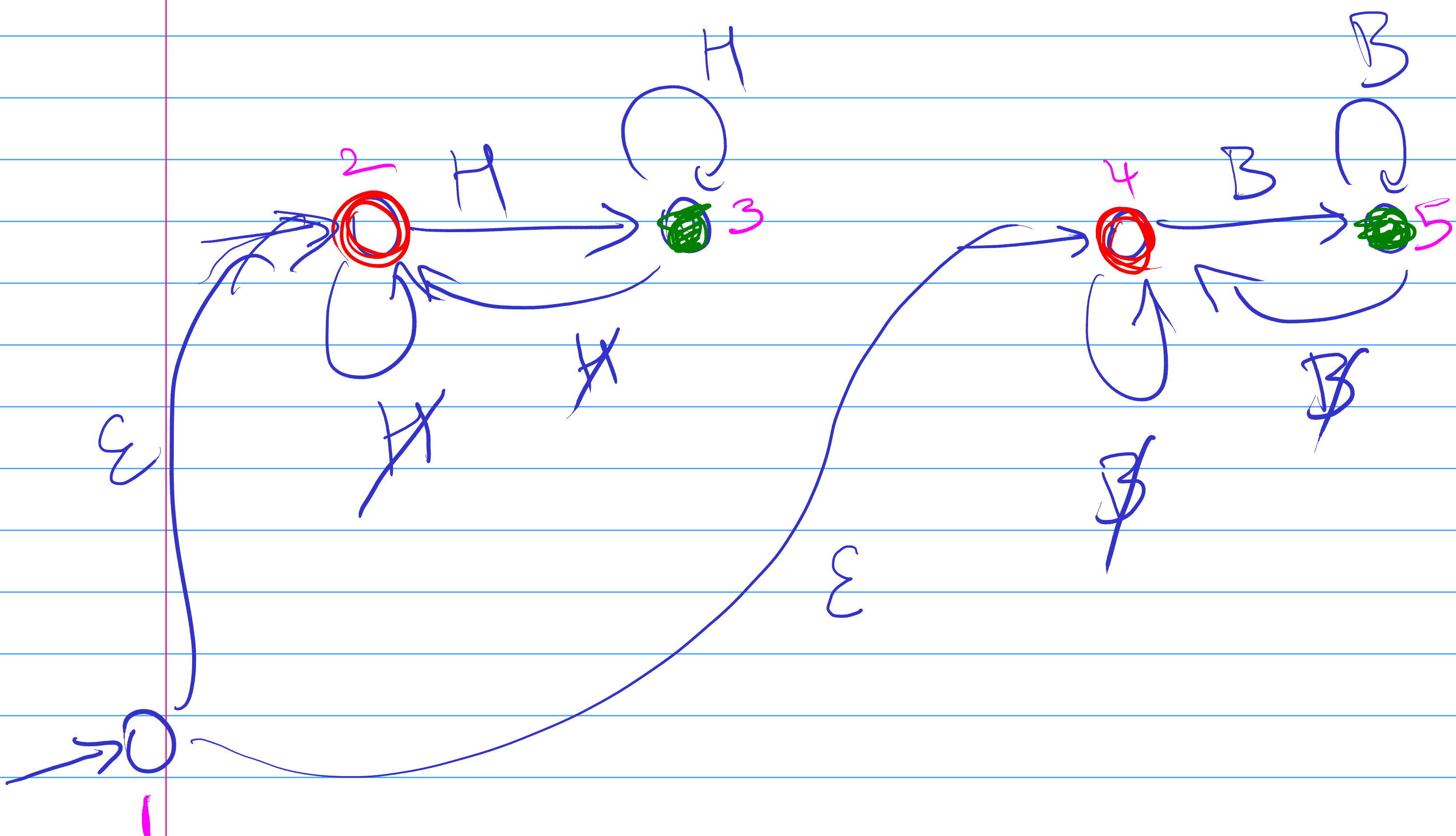
Question: Can we somehow play both games at once?



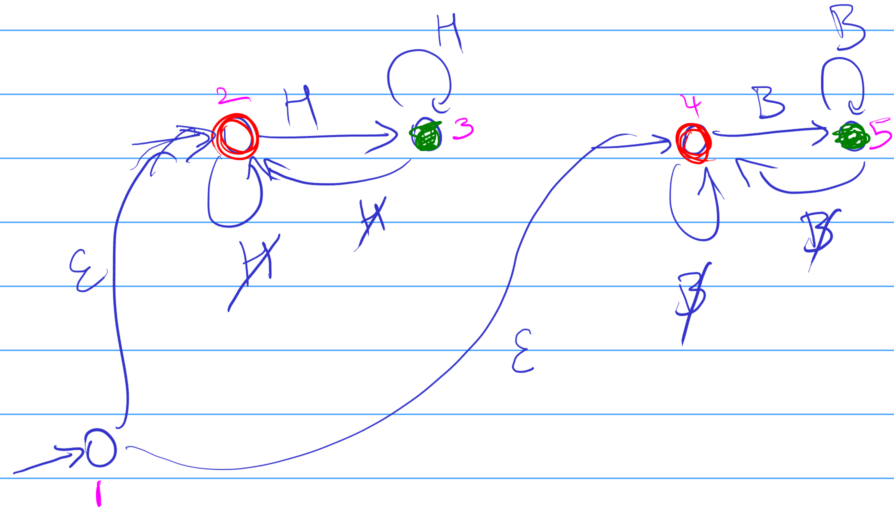
① Place token here at start

② Whenever token is on the state to the left, you get another token for free on the right.

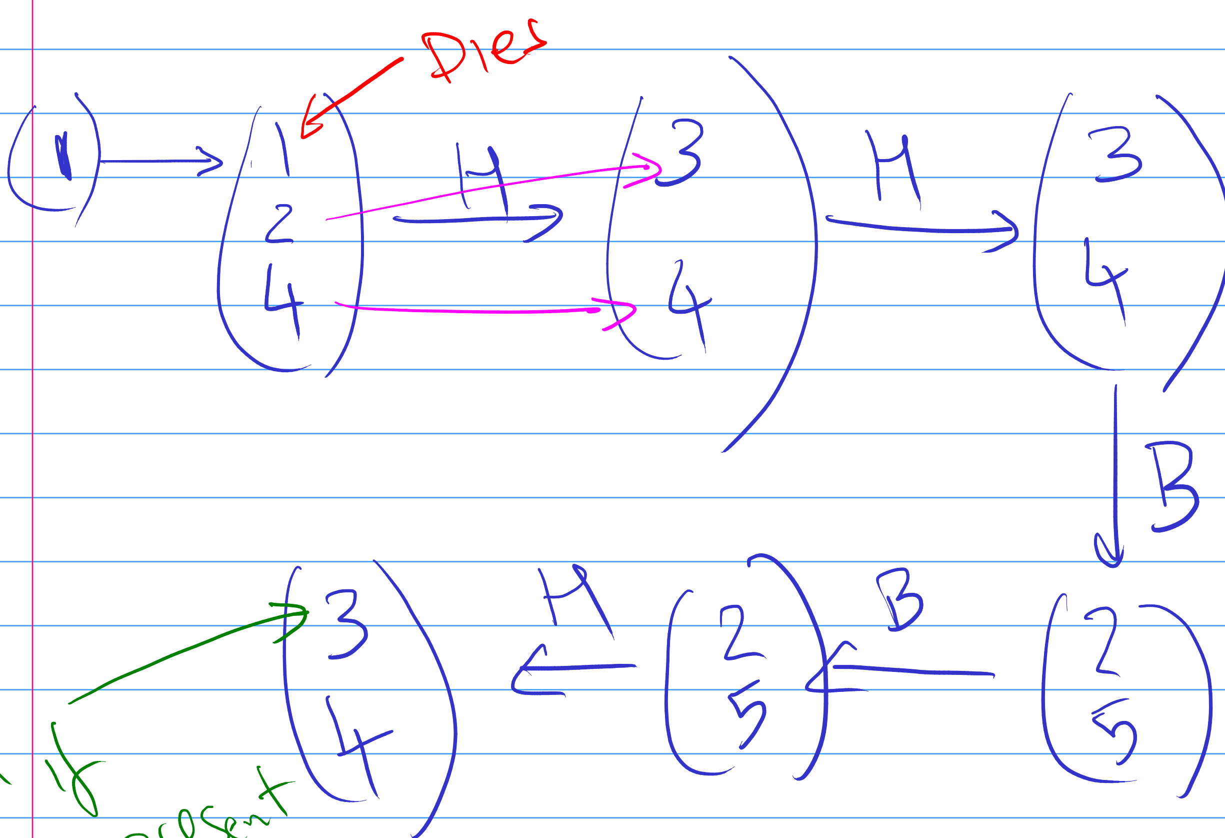
Ex : $\Sigma^{\lambda} H + \Sigma^{\lambda} B$



HHBBH

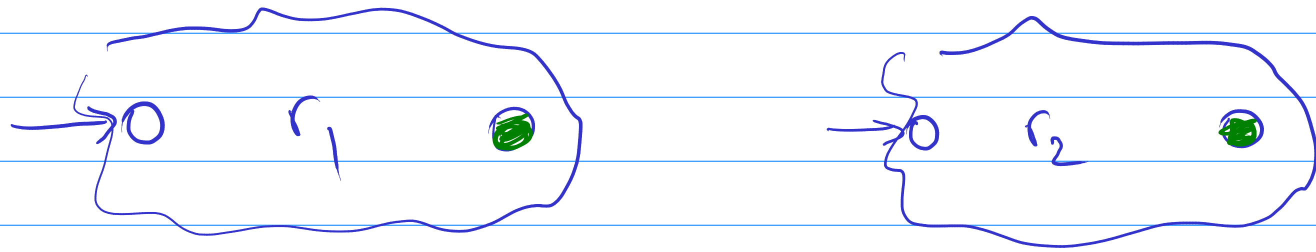


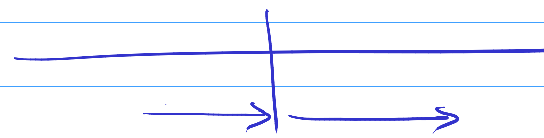
HHBBH



Win if token present in 3 so win!

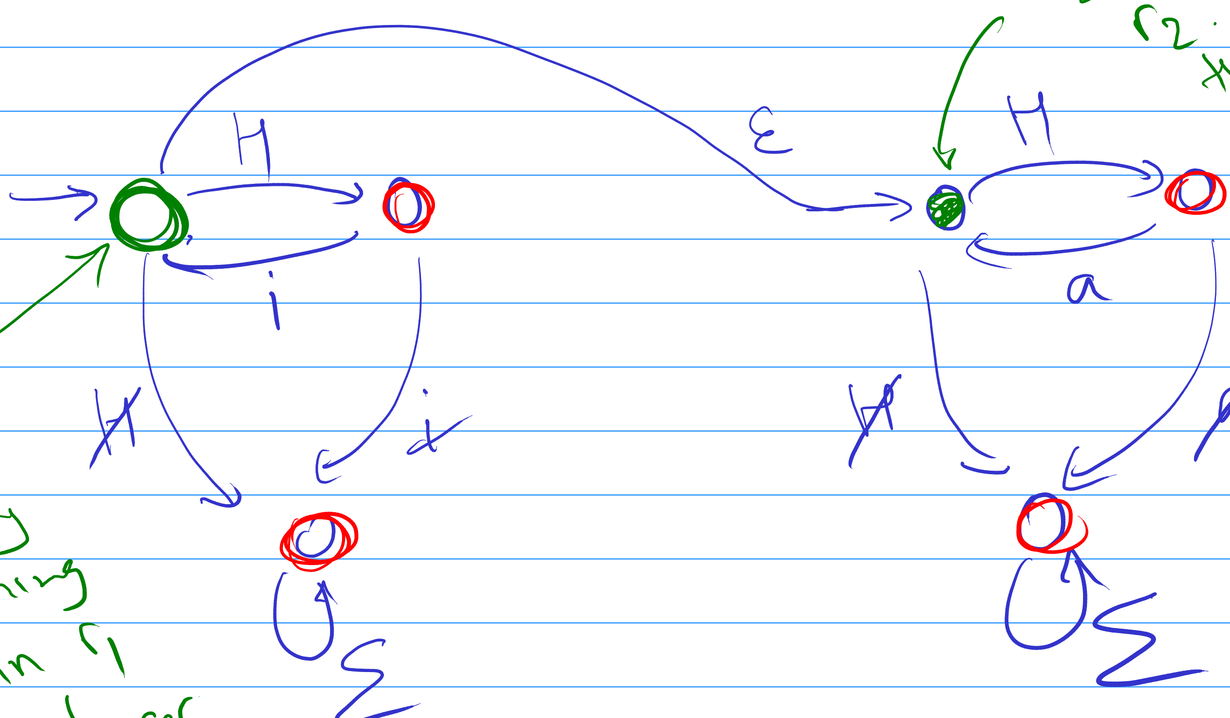
$\boxed{r_1 \cdot r_2}$





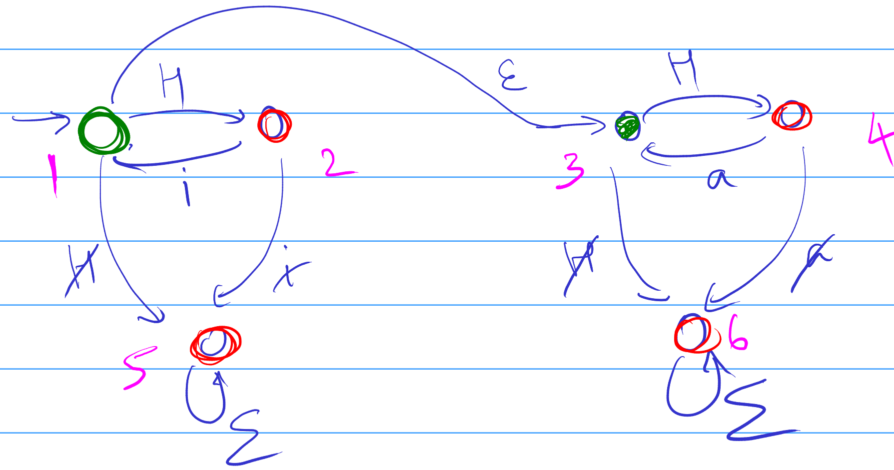
Ex: $(Hi)^* \cdot (Ha)^*$

Was the winning state in r_2. still the winning state.



Was originally a winning state in r_1 But no longer

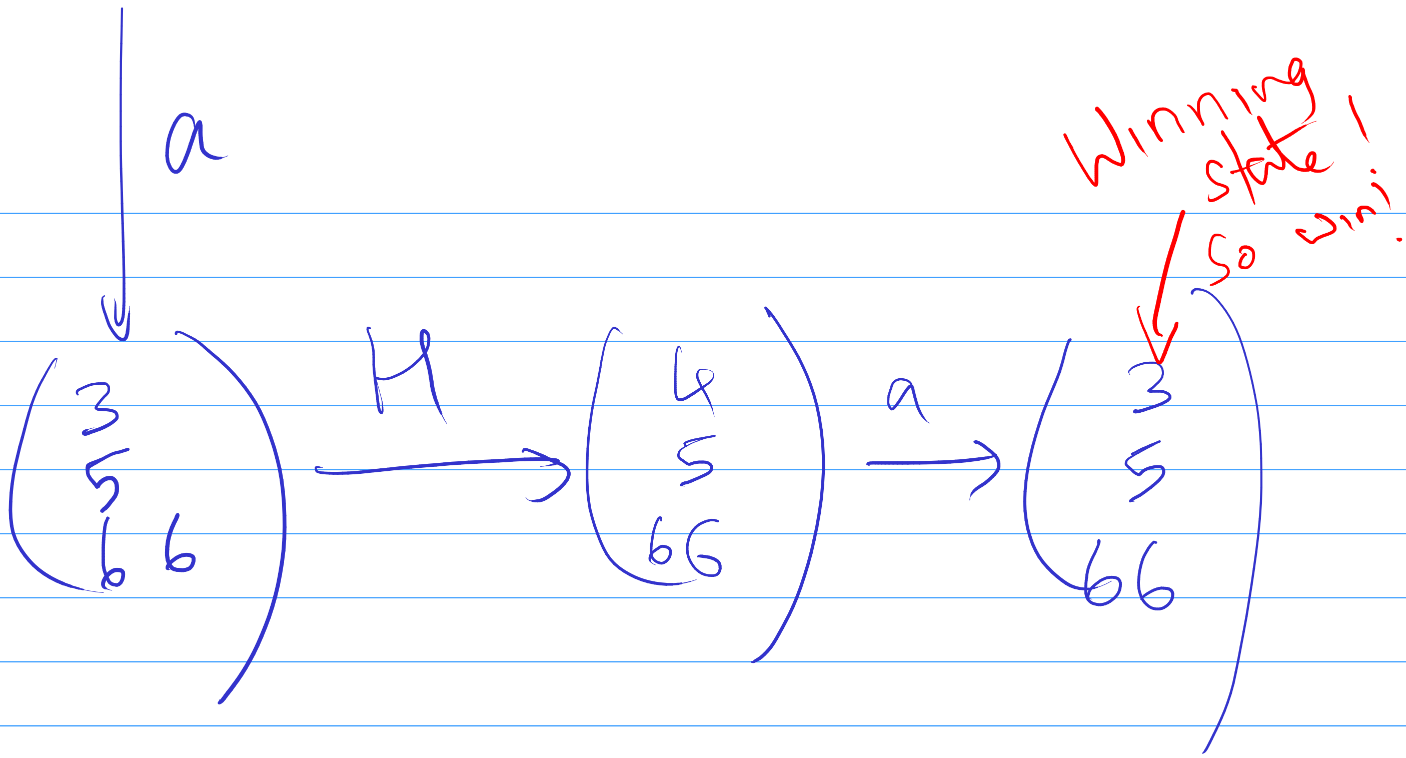
Ex: $(Hi)^* \cdot (Ha)^*$

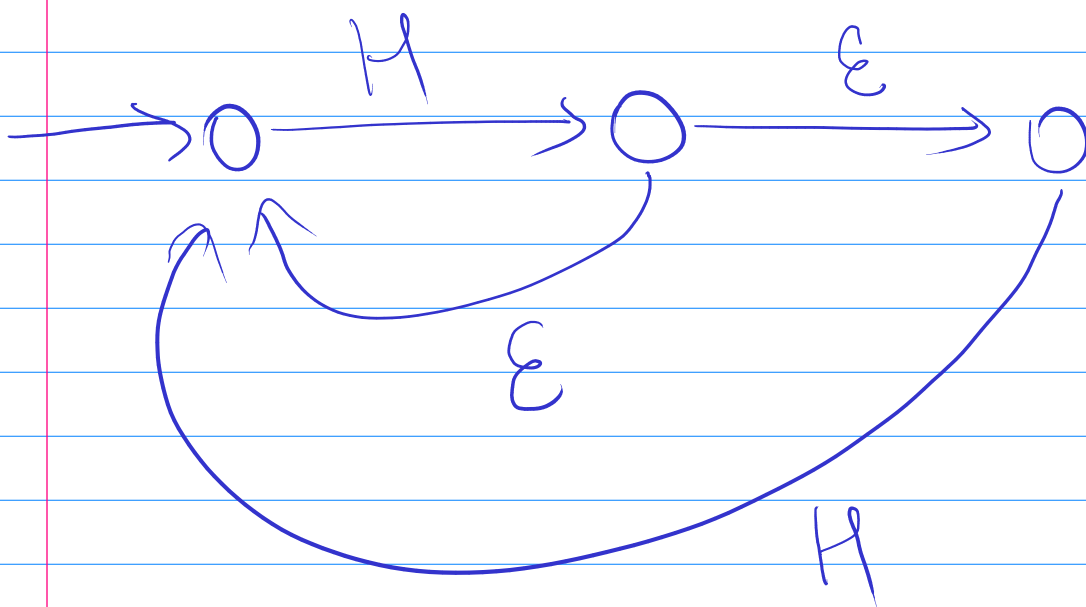


Hi Hi | Ha Ha

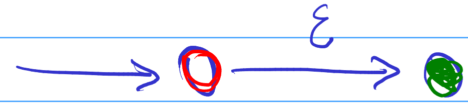
$$\begin{pmatrix}1\end{pmatrix} \longrightarrow \begin{pmatrix}1\\3\end{pmatrix} \xrightarrow{H} \begin{pmatrix}2\\4\end{pmatrix} \xrightarrow{i} \begin{pmatrix}1\\6\end{pmatrix} \longrightarrow \begin{pmatrix}1\\3\\6\end{pmatrix} \xrightarrow{H} \begin{pmatrix}2\\4\\6\end{pmatrix} \xrightarrow{i} \begin{pmatrix}1\\6\\6\end{pmatrix} \longrightarrow \begin{pmatrix}1\\3\\6\,6\end{pmatrix} \xrightarrow{H} \begin{pmatrix}2\\4\\6\,6\end{pmatrix}$$

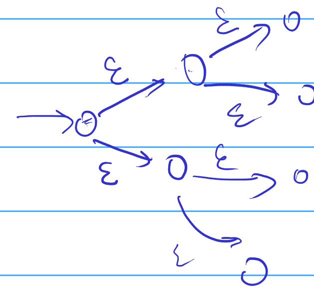
No free tokens


a
3
5
6 6
H
4
5
66
a
3
5
66
Winning state!
So win!

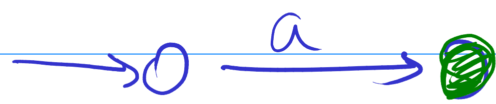

H
ε
ε
H

$\boxed{\varepsilon}$

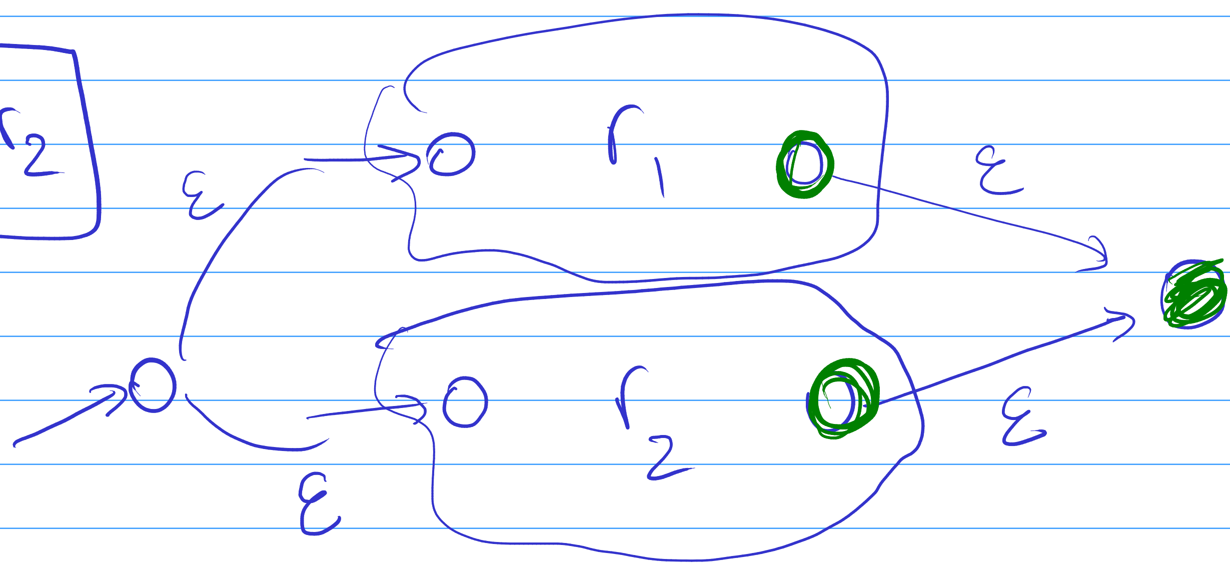




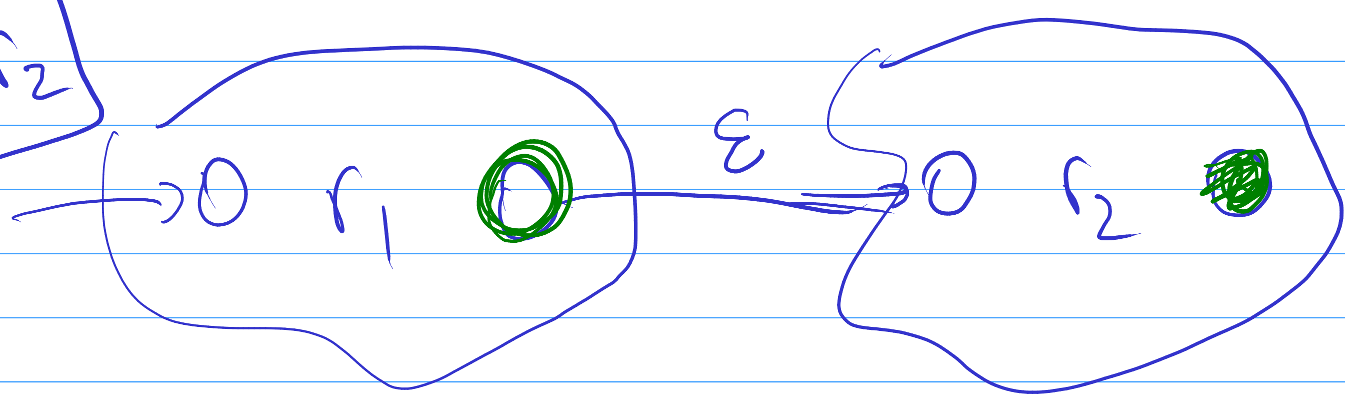
$\boxed{a}$



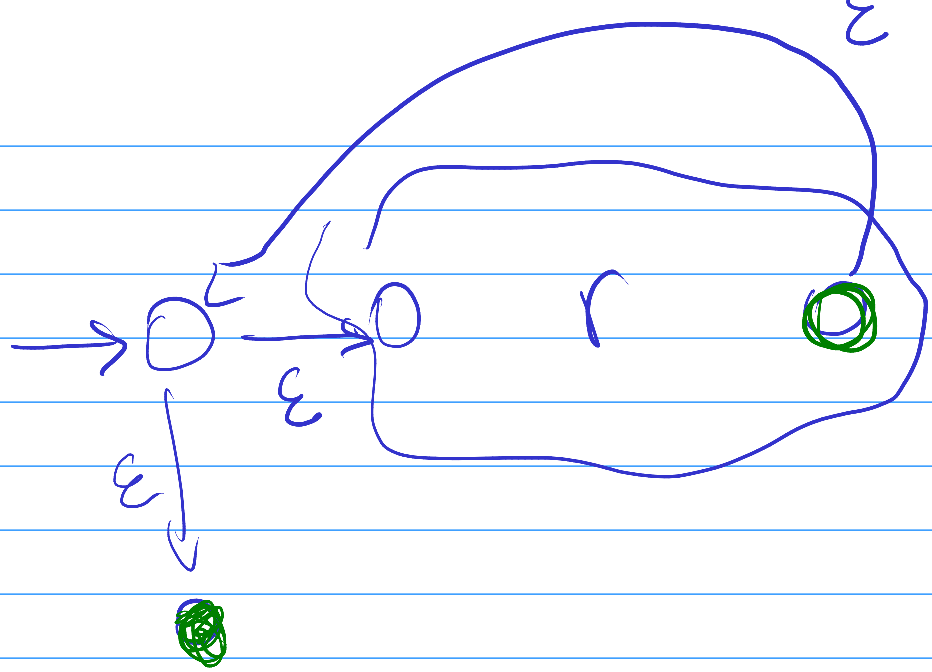
$\boxed{r_1 + r_2}$



$\boxed{r_1 \cdot r_2}$



r^*



$\varepsilon^* = \varepsilon$

State machine has no more than $O(|r|)$ states

"Board games"

Automata → Multiple tokens in play
Token promotion
Nondeterministic

Deterministic