

Stack size = 1 GB

$$= 1024 \times 1024 \times 1024 \text{ bytes}$$

Each call to sum must remember

- n (8 bytes)
- return address (8 bytes)

Notice that

$$\frac{1024 \times 1024 \times 1024}{33570783 \times (8+8) \times 2} \approx 1$$

Largest input n that
we can supply

unexplained.

Sum $n =$ if $n=0$ then 0
else $n + \text{sum}(n-1)$

int ans = 0

for ($i=0$; $i \leq n$; $i++$)

ans = ans + i

sum 5 $\Rightarrow 5 + \text{sum } 4$

$\Rightarrow 5 + (4 + \text{sum } 3)$

$\Rightarrow^* 5 + (4 + (3 + (2 + (1 + 0))))$

ans = 0

ans = ans + 1 = 1

ans = ans + 2 = 3

ans = ans + 3 = 6

ans = ans + 4 = 10

$((((0+1)+2)+3)+4)+5$

`Qcaml`

helper ans n

= if $n=0$ then ans
else helper (ans+n) (n-1)

Before fn. application

Before fn. application

The fn does not need to do

sum $n = n + \text{sum}(n-1)$

Recursive call

Addition happens
"afterwards"

Continuation

anything "after"
the recursive
call.

Empty continuation

int ans = 0

`C`

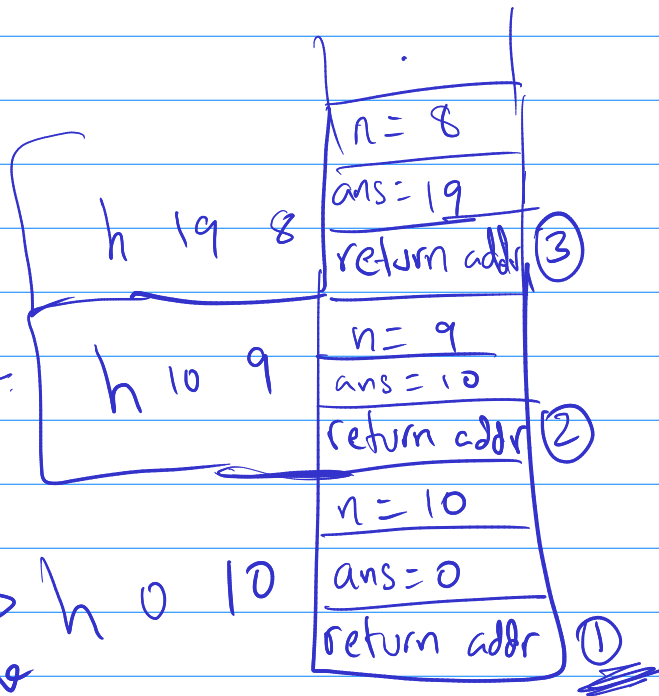
for (i:0 ; i ≤ n ; i++)

ans = ans + i

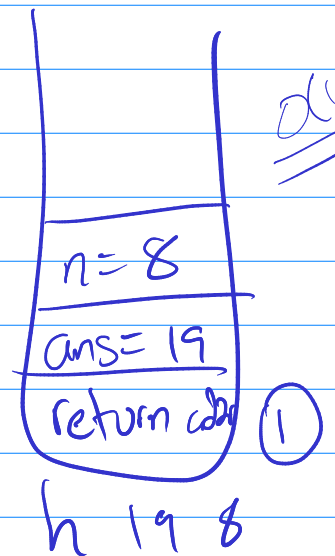
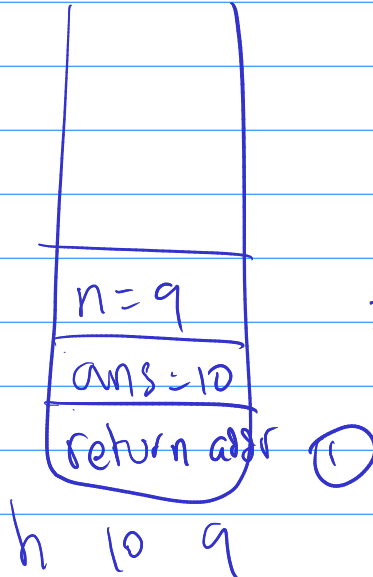
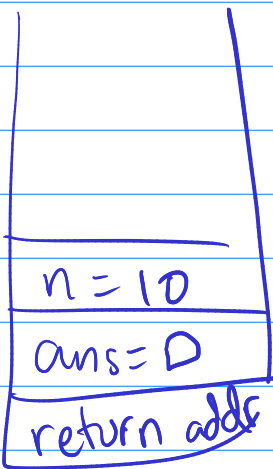
helper 0 10

Given that we have an empty continuation

Whatever value this fn returns will also be returned by this outside fn



$O(n)$



$O(n)$

$f\ x = \text{~~~~~} f(\text{---})$

Work to be done
after the recursive
call

"Continuation".

Optimization: If continuation is empty

Then don't allocate a new stack frame.

Tail call
optimization

Reuse

Q: Why no tail call optimization in ~~C++~~ JavaScript?

A1: Maybe C is too complicated?

A2: May gcc developers are stupid?

A3: Maybe they have good reasons not to?

tail call

tail recursion.

my rev [1 2 3 4 5]

(my rev [2 3 4 5]) @ [1]

my rev [1 2 3 4 5]

TODO list

~~my rev~~
h

[2 3 4 5]

[1]

↓ tl / pattern match

h

[3 4 5]

[2 1]

2 :: [1]

↓ tl / pr

h

[4 5]

[3 2 1]

3 :: [2 1]

↓ tl / pr

h

[5]

[4 3 2 1]

4 :: [3 2 1]

h

[]

[5 4 3 2 1]