Lecture 11

Wednesday, February 26, 2020 5:03 PM

- Today is the proposal deadline. (AoE)
- Today's plan
  - Finish discussion of HW1
    Q1, Q4, Q5 (P_6).
- Recap of DPLL
- Introduce clause learning
- Stretch goal
  - Introduce $\underbrace{\text{SMT}}_{\text{Satisfiability Modulo Theory}}$
  - Introduce theories:
    E, EUF
    LIA, Difference logic
    Arrays, Bitvectors

$\boxed{\text{Homeworks due on Friday the 6th, AoE.}}$

- Architecture of an SMT solver.

Q1: See updated notes from Lecture 10.

Q4 Distorted Hoare rule for assignments.

$$\overline{\{Q[e/x]\}\ x := e\ \{Q\}}$$

The weakest precondition of the assignment

$$x := e$$

wrt. the post condition Q is:

to replace every occurrence of x in Q with e.

$$\{z+2 > y+5\}\ x := z+2\ \{x > y+5\}$$

$z > y+3$

$\{true\}$
$x := y+3$
$\{x = y+3\}$

$\{y=8\}$
$x := y+z+9$
$\{y=8 \text{ and } x=y+z+9\}$

$\{true\}$
$x := y+z+9$
$\{x = y+z+9\}$

Is this true as a general principle?

N $\{true\}\ x := e\ \{x = e\}$

Is the following true as a general principle?

$\{P\}\ x := e\ \{P[x/e]\}$

S

0 | No | No |

po

Reference rule:

$\overline{\{Q[e/x]\}\ x := e\ \{Q\}}$

$\{x$

z

$\{x = y+9\}$
$z := y+9$

$\{x = y+8 \text{ and } y = z+9\}$
$w = y+8$

$z := y+9$

$\{x = z\}$

$w = y+8$

$\{x = w \text{ and } y = z+9\}$

trongest stcond:

=y+9 and =y+9}

$\{x = y+9\}$

$z := (y+9)+1$

$\{x = y+9\}$

$\{x = 1 \text{ and } e = 2\}$

$x := e$

$\{x = 1 \text{ and } x = 2\}$

Absurd.

Follow up: If P does not contain e, then is the following true as a general principle?

$$\frac{}{\{P\}\ x := e\ \{P[x/e]\}}$$

$$|||$$

$$\frac{}{\{P\}\ x := e\ \{P\}}$$

(In this specific case)

$\{P\}\ x := e\ \{P\}$ case)

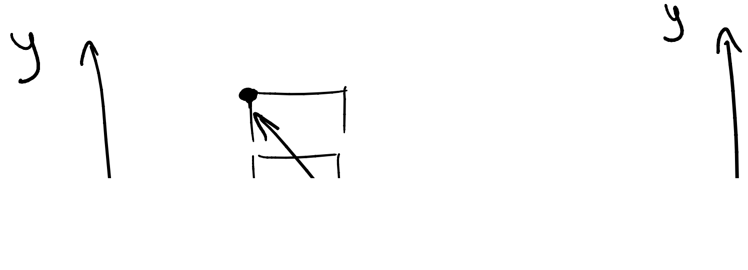
$\{a = b + c\}$	$\{x = 1\}$
$x := 3$	$x := 0$
$\{a = b + c\}$	$\{x = 1\}$
	Absurd.

Follow up 2: If x does not occur in P, then is the following true as a general principle?

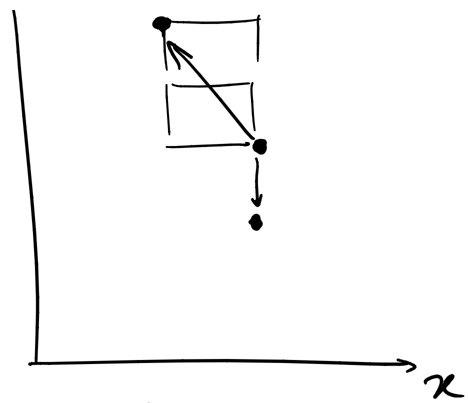
$$\{P\}\ x := e\ \{P[x/e]\}$$

Conjecture: This is valid.

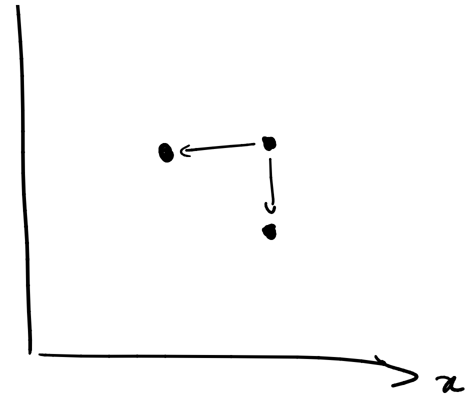
Q5: Proving termination



P



P_6



P_*

roof strategy

Find some fn:

① $f(x, y) \mapsto \mathbb{N}$

Show that if $x = x^*$ & $y = y^*$,

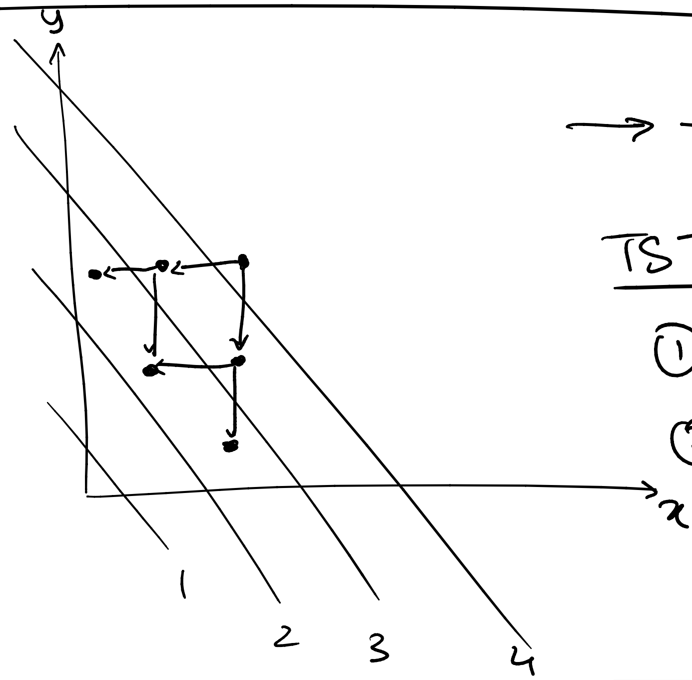
② then loop terminates in $f(x^*, y^*)$ iterations.

```
x := input
y := input
while (x>0 and y>0) {
    z := input
    if (z>0) {
        x := x-1
    } else {
        y := y-1
    }
}
```

For part ②, show that

- $b = \text{true} \Rightarrow f(x^*, y^*) > 0$, and
- $(x^*, y^*) \rightsquigarrow (\tilde{x}\ \tilde{y})$

$$f(x^*, y^*) \geq_+ f(\tilde{x}, \tilde{y})$$



Ranking fn.

$\rightarrow f(x, y) = x + y$

TST:

① $b = \text{true} \Rightarrow f(x,y) \geq_+ 0$

② $f(x,y) \geq_+ f(\tilde{x}, \tilde{y})$

① $b = \text{true} \equiv x > 0$ and $y > 0$

$$\Downarrow$$

$$x + y \geq_+ 0$$

$$\Downarrow$$

$$f(x, y) \geq_+ 0.$$

② Either $\tilde{x} = x - 1, \tilde{y} = y$) Case 1

or $\tilde{x} = x, \tilde{y} = y - 1$) Case 2

In case 1, show that

$$f(x, y) \geq_+ f(x-1\ y)$$

$$f(x, y) \underset{+}{\geq} f(x-1 \quad y)$$

$$\iff x+y \underset{+}{\geq} x-1+y$$

viz. Obvious

In case 2, show that

$$f(x, y) \underset{+}{\geq} f(x \quad y-1)$$

$$\iff x+y \underset{+}{\geq} x+y-1$$

viz. also obvious.