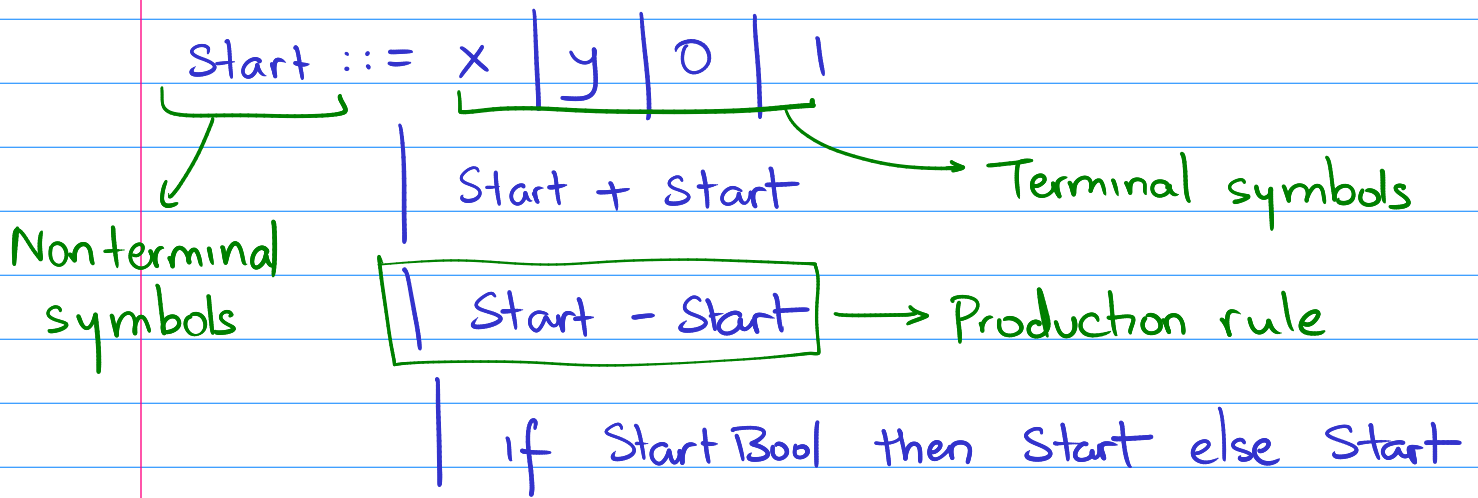


Lecture 13: Building a Program Synthesizer

Announcements

- Homework 1
-



$\text{Start Bool} ::= \text{Start} \leq \text{Start}$

Start Bool and Start Bool

not Start Bool

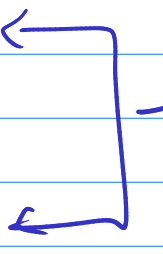
The SyGuS problem

$$\exists f. \forall \vec{x}. \varphi(\vec{x}, f) ?$$

\Downarrow
 \Downarrow

Simpler problem

How to check if a given function \underline{f} satisfies the specification?

$$\forall \vec{x}. \varphi(\vec{x}, f) ? \leftarrow$$
$$\exists \vec{x}. \neg \varphi(\vec{x}, f) ? \leftarrow$$


Ex: $\exists f. \forall x \in \mathbb{Z} \cdot x \leq f(x) \text{ and } f(x) \leq x+5?$

Check $f(x) = x + 9$.

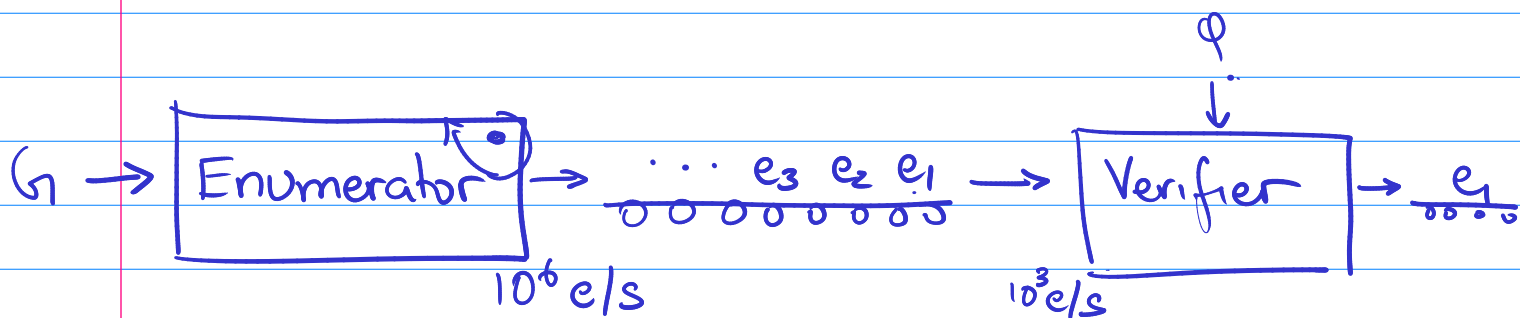
$\forall x \in \mathbb{Z} \quad x \leq x+9 \text{ and } x+9 \leq x+5?$

$\Leftrightarrow \neg \left(\exists x \in \mathbb{Z} \underbrace{\neg (x \leq x+9 \text{ and } x+9 \leq x+5)} \right)$

Essentially, we ask \exists :

"Can you give us a counterexample x
where f does not work?"

The SyGuS Factory Version 0



CNF

$$e_1 = x$$

$$e_2 = \neg y$$

$$e_3 = \text{not } x$$

$$e_4 = \text{not } y$$

$$e_5 = x \text{ and } y$$

$$e_6 = x \text{ and } (\text{not } y)$$

⋮

$$\exists \vec{x}. \neg \varphi(\vec{x}, c)?$$

If yes, done

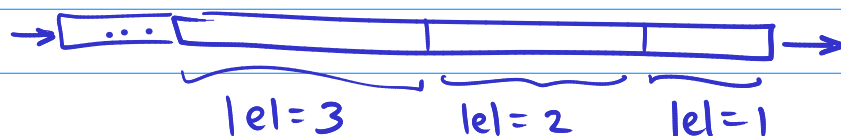
Otherwise, drop c .

SMT

Note: Enumerate expressions in order of increasing size.

Otherwise, weird stuff happens

Also, assume that the problem is solvable.



- Enumerate all expressions of size 1

$$1 \ 0 \ x \ y$$

:

- Enumerate all expressions of size 10

→ Start ::= Start + Start | → Start ::= Start - Start

$$9 = ① + ⑧$$

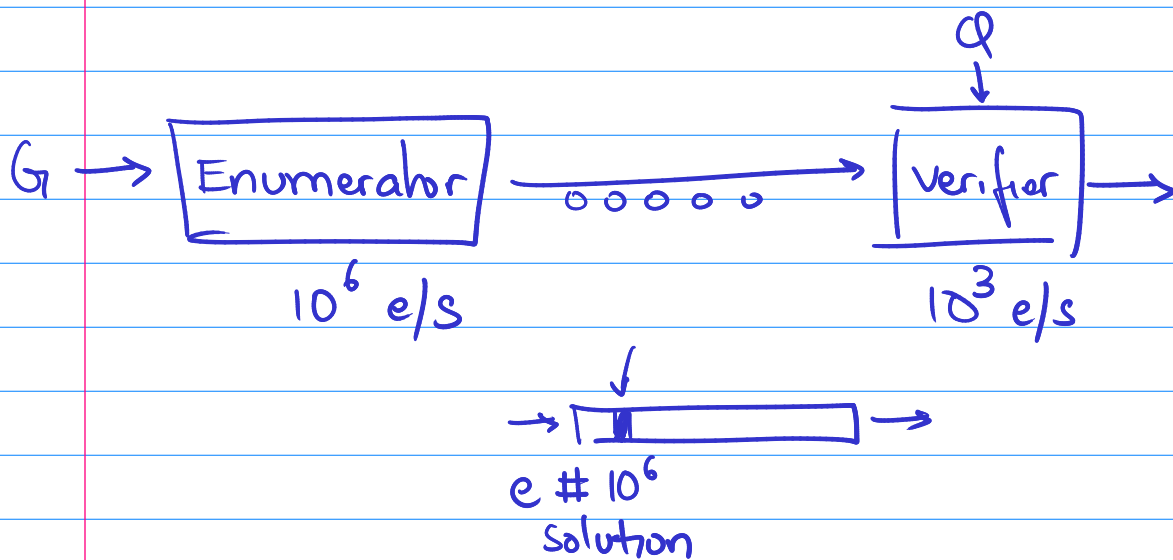
$$= 2 + 7$$

$$= 3 + 6$$

$$= \dots$$

$$= 8 + 1$$

Analyzing SyGuS Factory Version 0



Time for discovery = 1001 seconds.

SyGuS Factory Version 1.0

$$\exists f \in G \quad \forall x \in \mathbb{N} \quad \varphi(x, f) ?$$

$$\exists f \in G \quad \varphi(0, f) \text{ and } \varphi(1, f) \text{ and } \varphi(2, f)$$

and ...

Pick $C = \{1, 8, 14, 217\}$ ← "Counterexamples"

If f satisfies the spec, then it also satisfies

$$\forall x \in C \quad \varphi(x, f).$$

$\varphi(1 f)$ and $\varphi(8 f)$ and
 $\varphi(14 f)$ and $\varphi(217 f)$

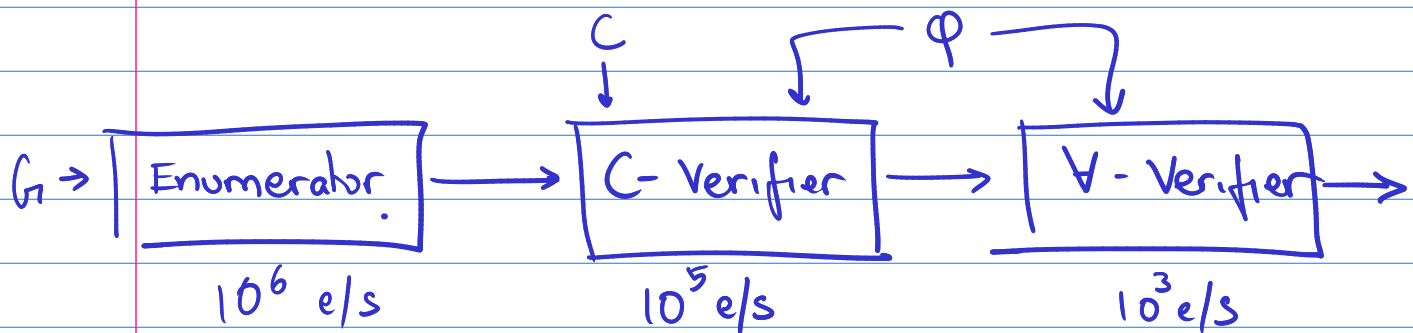
Ex: $\exists f. \forall x \in \mathbb{N} \quad \underbrace{x \leq f(x) \text{ and } f(x) \leq x+1}$

$$C = \{16, 18\} \subseteq \mathbb{N}$$

$$\forall x \in \{16, 18\}$$

$$\left(16 \leq f(16) \text{ and } f(16) \leq 16+1 \right)$$

$$\text{and } \left(18 \leq f(18) \text{ and } f(18) \leq 18+1 \right)$$



$$\frac{10^6}{10^6} + \frac{10^6}{10^5} + \frac{10^3}{10^3} = 12 \text{ seconds}$$



$e\# 10^6$ $e\# 1$
 Solution

1000 false positives w.r.t C.

Question: How to choose c ?

$$\neg(\exists x. \neg \varphi(x, f)) \Leftrightarrow \forall x. \varphi(x, f).$$

If candidate function f fails,

we have a concrete point at which it fails.

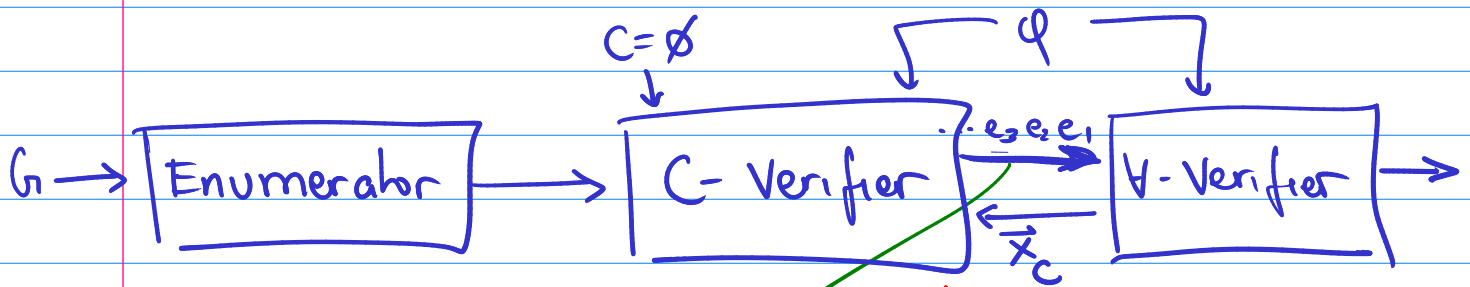
Ex: $\exists f. \forall x. f(x) \leq x+3$

Try $f(x) = x+8$

$$\exists x. x+8 \not\leq x+3$$

SMT: Consider $x=7$ \rightsquigarrow This is a concrete reason for the candidate function to fail
"Counterexample".

SyGuS Factory Version 2.0

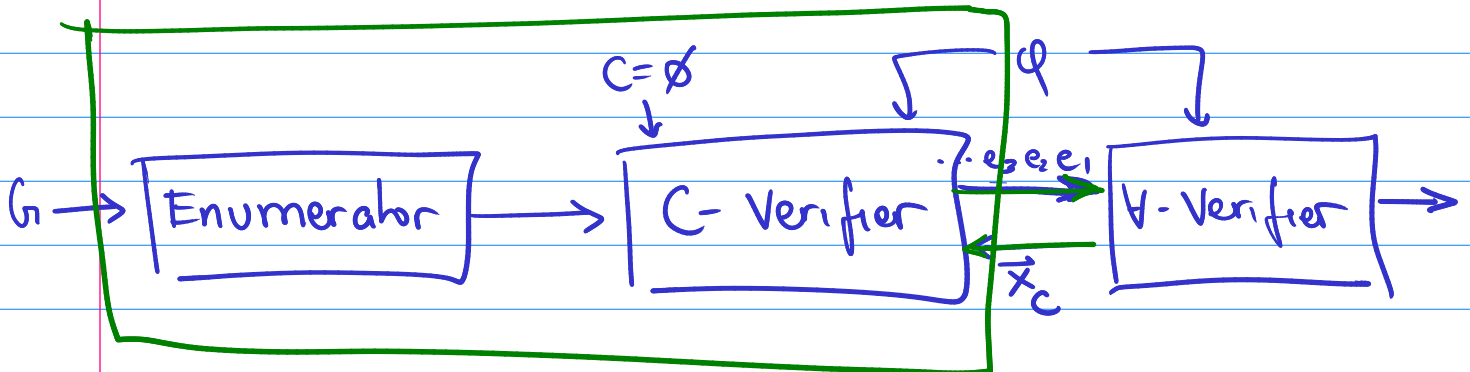


- No pair of functions is wrong for the same "reason".

C-Verifier adds \bar{x}_C into C .

- Each expression is discovering something new about the function to be synthesized.

Counter-Example Guided Inductive Synthesis (CEGIS)



"Learner"

