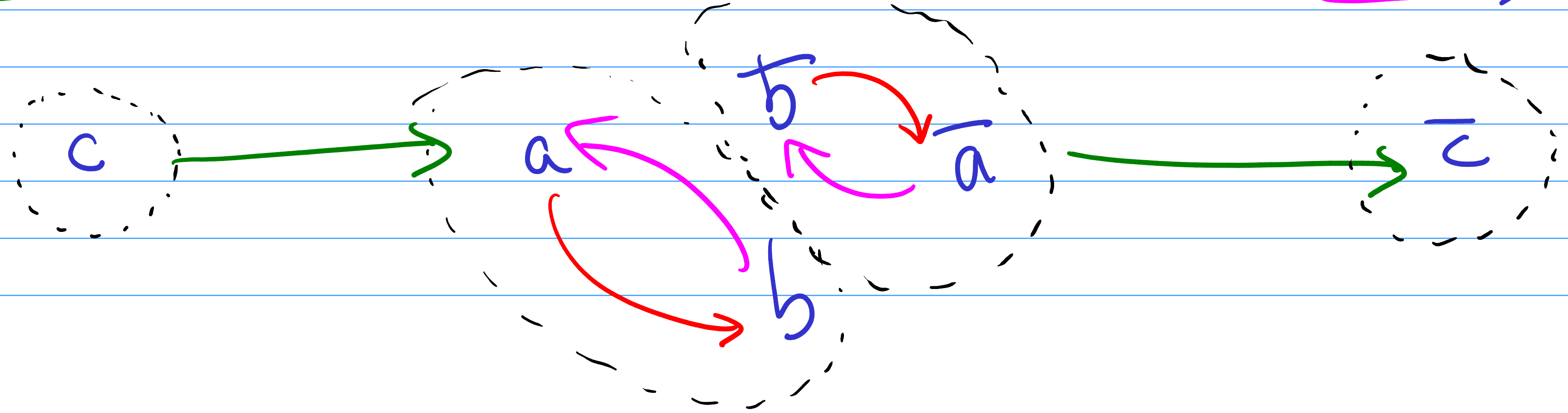
$\overline{\varphi} \vee \varphi_2 \equiv \varphi_1 \Rightarrow \varphi_2$

Polynomial Time Algorithm for 2-SAT

$(\bar{c} \vee a)$ and $(\bar{a} \vee b)$ and $(a \vee \bar{b})$

$(c \Rightarrow a)$ and $(a \Rightarrow b)$ and $(b \Rightarrow a)$



<u>Claim 1</u> : If v & $\bar{v}$ occur in the same SCC in the implication graph, then φ is Unsat.

<u>Claim 2</u> : If all SCCs are "good", ~~then~~ variable assignment can never get stuck.

$(\cancel{x1} \lor x4)$
and $(\cancel{x1} \lor \neg x3 \lor \neg x8)$
and $(\cancel{x1} \lor x8 \lor x12)$
and $(x2 \lor x11)$
and $(\neg x7 \lor \neg x3 \lor x9)$
and $(\neg x7 \lor x8 \lor \neg x9)$
and $(x7 \lor x8 \lor \neg x10)$
and $(x7 \lor x10 \lor \neg x12)$

Choice #1

$\neg x_1$

$\downarrow$

x_4

$\cancel{(x1 \lor x4)}$
and $(\cancel{x1} \lor \neg x3 \lor \neg x8)$
and $(\cancel{x1} \lor x8 \lor x12)$
and $(x2 \lor x11)$
and $(\neg x7 \lor \neg x3 \lor x9)$
and $(\neg x7 \lor x8 \lor \neg x9)$
and $(x7 \lor x8 \lor \neg x10)$
and $(x7 \lor x10 \lor \neg x12)$

$\cancel{(x1 \lor x4)}$

and $(\cancel{x1} \lor \neg x3 \lor \neg x8)$

and $(\cancel{x1} \lor x8 \lor x12)$

and $(x2 \lor x11)$

and $(\neg x7 \lor \neg x3 \lor x9)$

and $(\neg x7 \lor x8 \lor \neg x9)$

and $(x7 \lor x8 \lor \neg x10)$

and $(x7 \lor x10 \lor \neg x12)$

Choice #2

$$x_3 \to \overline{x_8} \to x_{12}$$

$\cancel{(x1 \lor x4)}$

$\cancel{\text{and } (x1 \lor \neg x3 \lor \neg x8)}$

$\cancel{\text{and } (x1 \lor x8 \lor x12)}$

and $(x2 \lor x11)$

and $(\neg x7 \lor \cancel{\neg x3} \lor x9)$

and $(\neg x7 \lor \cancel{x8} \lor \neg x9)$

and $(x7 \lor \cancel{x8} \lor \neg x10)$

and $(x7 \lor x10 \lor \cancel{\neg x12})$

$\cancel{(x1 \lor x4)}$
and $\cancel{(x1 \lor \text{[illegible]} \lor \neg x8)}$
and $\cancel{(x1 \lor x8 \lor x12)}$
and $(x2 \lor x11)$
and $(\neg x7 \lor \cancel{\neg x8} \lor x9)$
and $(\neg x7 \lor \cancel{x8} \lor \neg x9)$
and $(x7 \lor \cancel{x8} \lor \neg x10)$
and $(x7 \lor x10 \lor \cancel{\neg x12})$

Choice #3

$\overline{x_2}$

$\downarrow$

x_{11}

$\cancel{(x1 \lor x4)}$
and $\cancel{(x1 \lor \text{[illegible]} \lor \neg x8)}$
and $\cancel{(x1 \lor x8 \lor x12)}$
and $\cancel{(x2 \lor x11)}$
and $(\neg x7 \lor \cancel{\neg x8} \lor x9)$
and $(\neg x7 \lor \cancel{x8} \lor \neg x9)$
and $(x7 \lor \cancel{x8} \lor \neg x10)$
and $(x7 \lor x10 \lor \neg x12)$

Choice #4

~~(x1 ∨ x4)~~
and ~~(x1 ∨ [illegible] ∨ ¬x8)~~
and ~~(x1 ∨ x8 ∨ x12)~~
and ~~(x2 ∨ x11)~~
and (¬x7 ∨ ~~¬x8~~ ∨ x9)
and (¬x7 ∨ ~~x8~~ ∨ ¬x9)
and (x7 ∨ ~~x8~~ ∨ ¬x10)
and (x7 ∨ x10 ∨ ¬x12)

$$x_7 \longrightarrow x_9,\ \overline{x_9}$$

Conflict!

~~(x1 ∨ x4)~~
and ~~(x1 ∨ [illegible] ∨ ¬x8)~~
and ~~(x1 ∨ x8 ∨ x12)~~
and ~~(x2 ∨ x11)~~
and (~~¬x7~~ ∨ ~~¬x8~~ ∨ x9)
and (~~¬x7~~ ∨ ~~x8~~ ∨ ¬x9)
and (x7 ∨ ~~x8~~ ∨ ¬x10)
and (x7 ∨ x10 ∨ ¬x12)

Conflict!

$\bar{x}_1$ x_3 $\bar{x}_2$ x_7

$x_1 \vee \bar{x}_3 \vee \bar{x}_7$

x_1

F

$x_4 \mapsto T$

x_3

T

$\bar{x}_8$ x_{12} $\bar{x}_7$

x_2

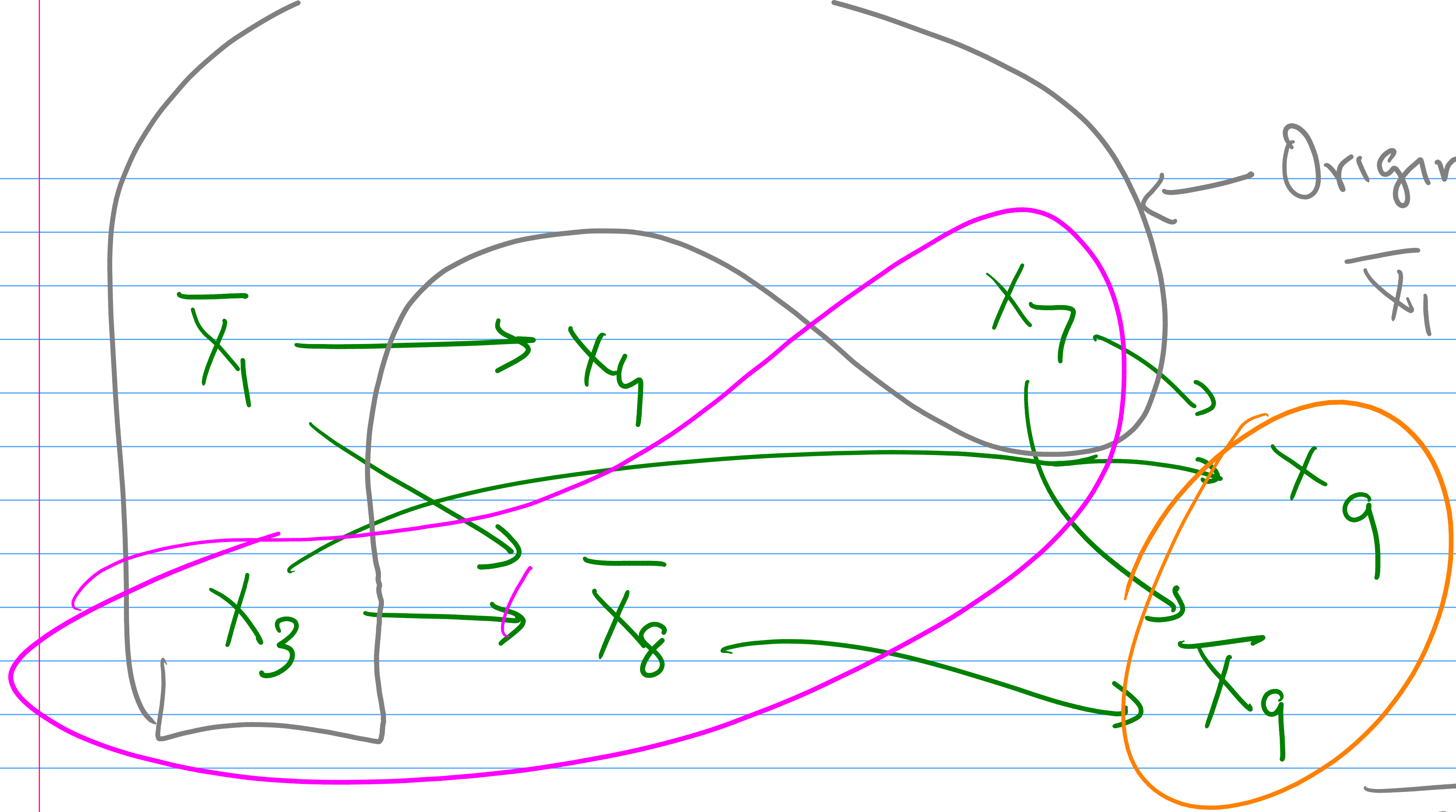
F

x_{11}

x_7

T

conflict



Original sin

$\overline{x_1} \wedge x_3 \wedge x_7$

$\overline{x_2} \longrightarrow x_{11}$

Moral ①: $\overline{\overline{x_1} \wedge x_3 \wedge x_7}$

$x_1 \vee \overline{x_3} \vee \overline{x_7}$

Moral ②: $\overline{x_3 \wedge \overline{x_8} \wedge x_7}$

$\overline{x_3} \vee x_8 \vee \overline{x_7}$

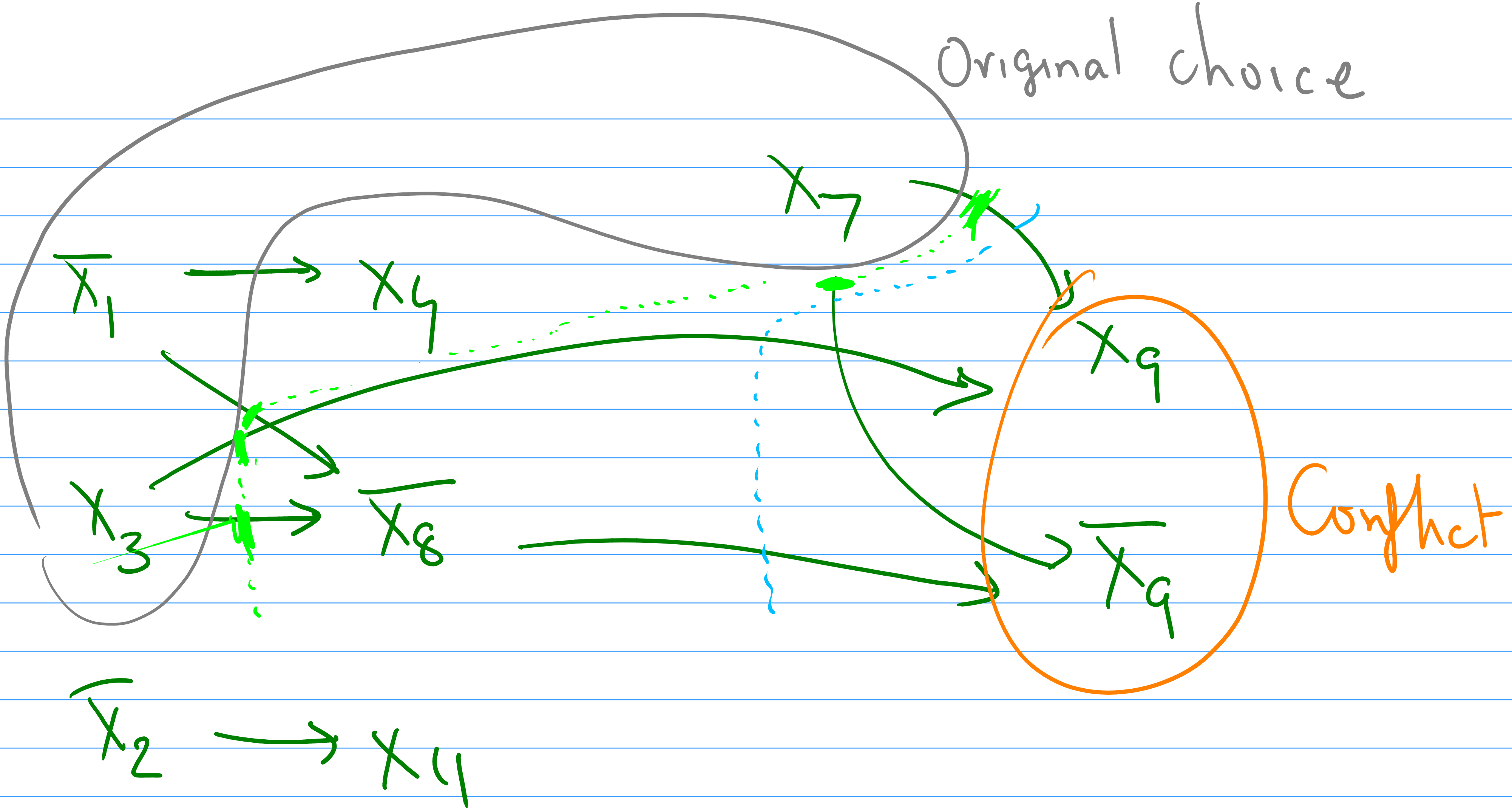
Lemma

(x1 ∨ x4)
and (x1 ∨ ¬x3 ∨ ¬x8)
and (x1 ∨ x8 ∨ x12)
and (x2 ∨ x11)
and (¬x7 ∨ ¬x3 ∨ x9)
and (¬x7 ∨ x8 ∨ ¬x9)
and (x7 ∨ x8 ∨ ¬x10)
and (x7 ∨ x10 ∨ ¬x12)

$$\Longrightarrow x_1 \vee \overline{x_3} \vee \overline{x_7}$$

So: Search for an assignment which satisfies

(x1 ∨ x4)
and (x1 ∨ ¬x3 ∨ ¬x8)
and (x1 ∨ x8 ∨ x12)
and (x2 ∨ x11)
and (¬x7 ∨ ¬x3 ∨ x9)
and (¬x7 ∨ x8 ∨ ¬x9)
and (x7 ∨ x8 ∨ ¬x10)
and (x7 ∨ x10 ∨ ¬x12)
and $(x_1 \vee \overline{x_3} \vee \overline{x_7})$


Original choice
$\bar{x}_1$
x_4
x_7
x_3
$\bar{x}_8$
x_9
$\bar{x}_9$
Conflict
$\bar{x}_2$
x_{11}

Watched literals

Decision heuristics

- Dynamic Largest Individual Sum
  Pick variable which satisfies most remaining clauses
- VSIDS : Variable state independent decaying sum

VSIDS

For each variable v, c_v = # of clauses with v

$c_{\bar{v}}$ = # of clauses with $\bar{v}$

Pick variable/polarity with greatest $c_{v/\bar{v}}$.

v $\pm v$

Periodically, divide all counters by 2.

$(\cancel{x1} \lor x4)$
and $(\cancel{x1} \lor \neg x3 \lor \neg x8)$
and $(\cancel{x1} \lor x8 \lor x12)$
and $(x2 \lor x11)$
and $(\neg x7 \lor \neg x3 \lor x9)$
and $(\neg x7 \lor x8 \lor \neg x9)$
and $(x7 \lor x8 \lor \neg x10)$
and $(x7 \lor x10 \lor \neg x12)$

$x_1 : 3$

$\bar{x}_1 : 0$

$x_2 : 1$

$\bar{x}_2 : 0$

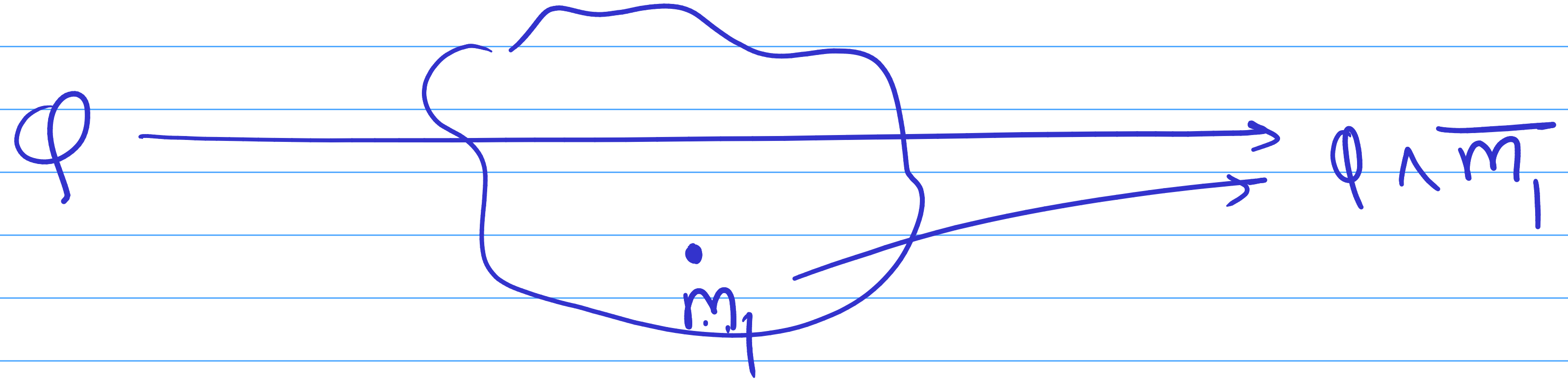
$x_3 : 0$

$\bar{x}_3 : 2 \to 1$

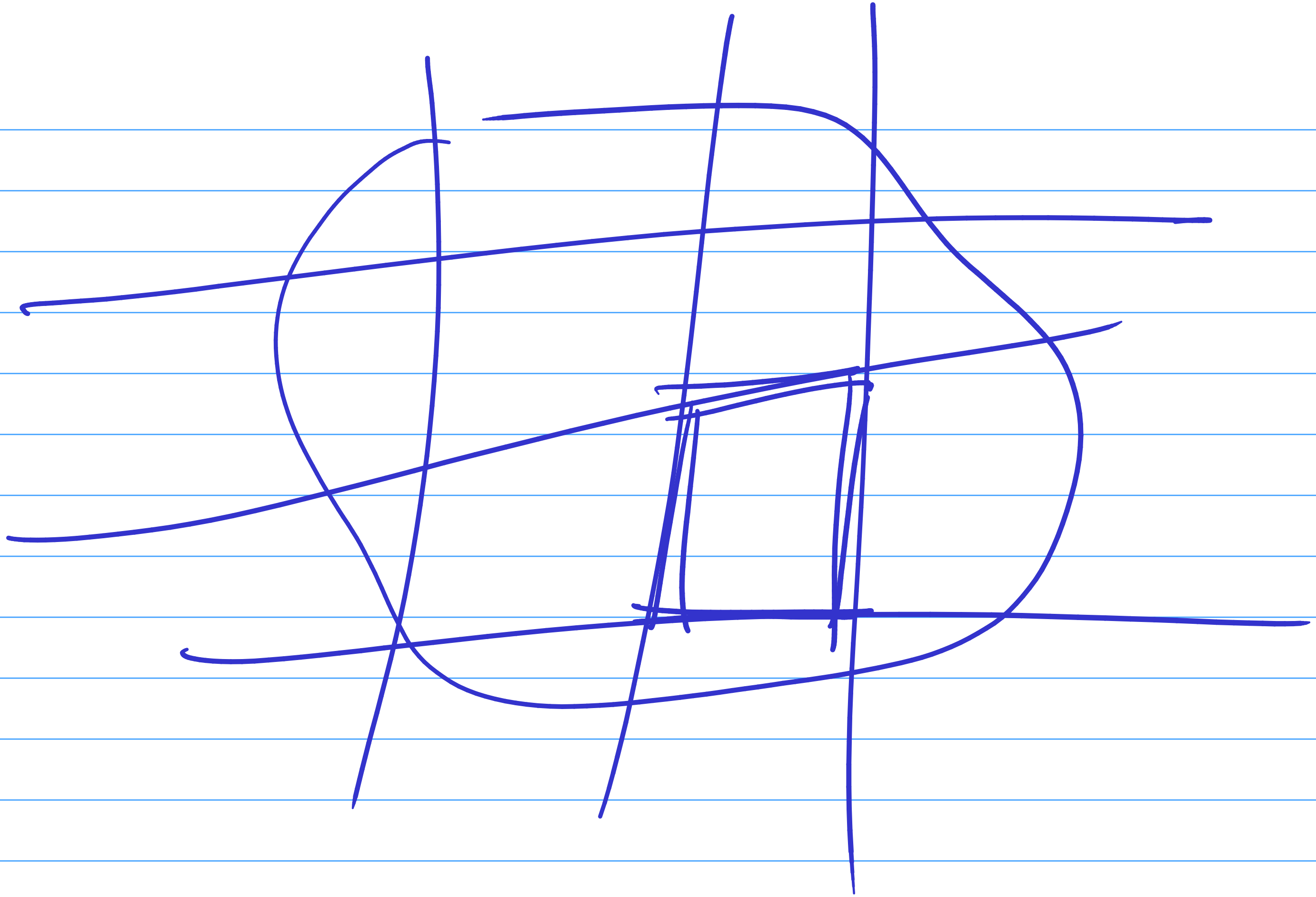
$\vdots$

$x_4 : 1 \to 0$

$\bar{x}_4 : 0$



$$
\begin{tikzcd}
\varphi \arrow[r] \arrow[d] & \varphi \wedge \overline{m_1} \arrow[r] \arrow[d] & \varphi \wedge \overline{m_1} \wedge \overline{m_2} \arrow[r] \arrow[d] & \cdots \\
m_1 & m_2 & \vdots &
\end{tikzcd}
$$



Satisfiability Modulo Theories

Boolean values $\longrightarrow$ Integers, arrays, strings, ...

$a \wedge (b \vee c)$

v_1 and v_2
and $(v_3$ or $v_4)$
and v_5

$x \geq 10$ and $y = x + x$
and $(y \leq 3$ or $y \geq 100)$
and $x \leq 15$

DPLL $\longrightarrow$ DPLL(T) $\longrightarrow$ Theory combination

CDCL

Nelson-Oppen

IDL RA

UF