1. Paper presentations on Feb 8

Model counting, ML-enabled decision heuristics

QBF solvers

WalkSAT

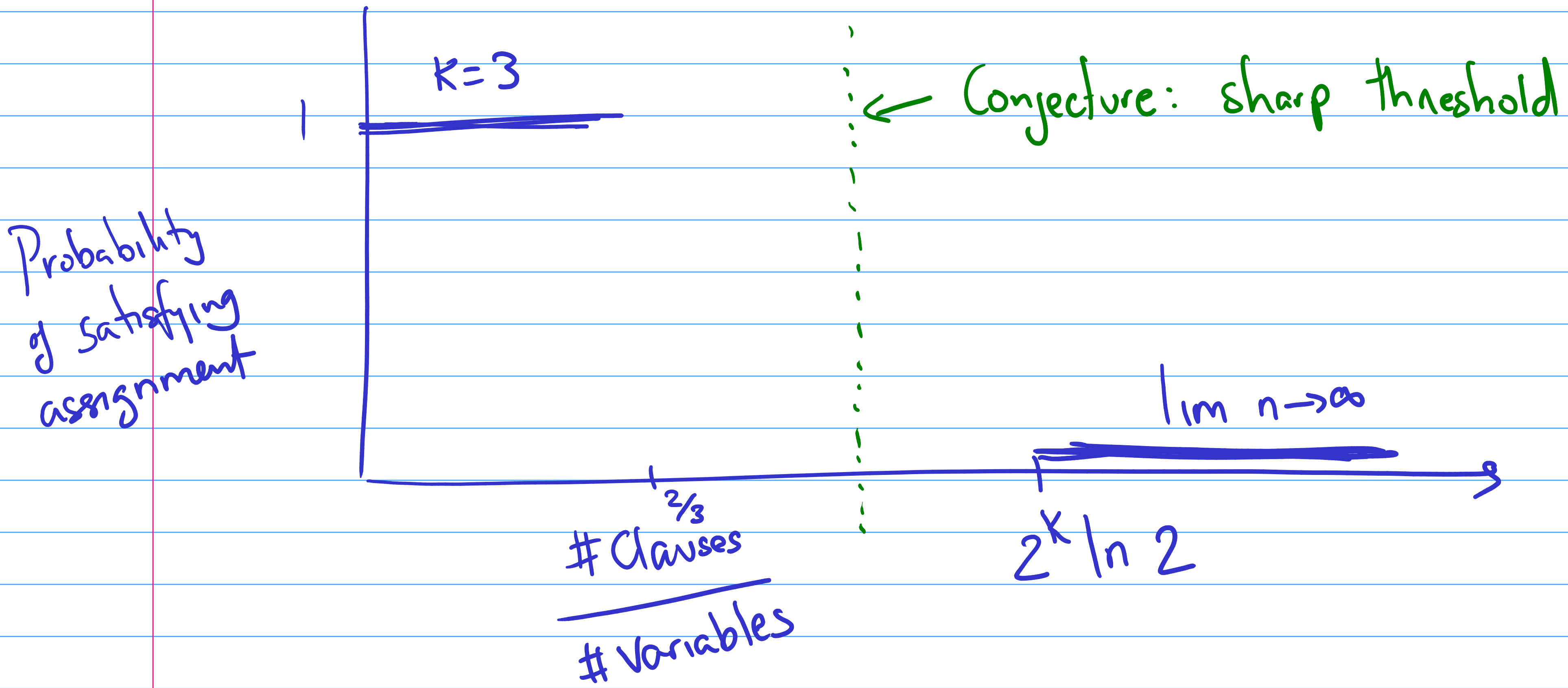
1. Guess an assignment
2. Repeat until assignment $\models \varphi$: "Models"
   - Pick an unsatisfied clause, uniformly at random
     $l_1 \vee l_2 \vee \cdots \vee l_k$
   - Pick a literal, uniformly at random
   - Flip it.

Random - CNF

① Fix n variables, clauses of length K. ← $\binom{n}{k} 2^k$ possible clauses

② Decide that formula has m clauses:

Sample clause (with replacement?).


k=3
1
Probability of satisfying assignment
Conjecture: sharp threshold
lim n→∞
2/3
Clauses / # Variables
2^k ln 2

Erdős–Rényi Graphs $G(n,p)$

For each pair (u, v) of vertices (undirected) with prob. p, insert edge (u, v).

Disconnected almost surely, if $p < \frac{1}{n}$

Many vertices in same SCC, if $p > \frac{1}{n}$.

Satisfiability Modulo Theory

```
int  x1  x2  x3  := input();

if (x1 ≠ x2)
    if (x2 = x3)
        if (x3 = x1)
            crash!
```

$\exists\, x_1\ x_2\ x_2$ st. $(x_1 \neq x_2)\ \&\ x_2 = x_3\ \&$
$x_3 = x_1$?

```
int i = j
arr[i] := 10;
if (arr[j] < 8) crash!
```

"Theory"

DPLL / CDCL $\longrightarrow$ DPLL(T) $\longrightarrow$ Theory combination

"Nelson-Oppen"

"Theory solver"

A solver for Linear Real Arithmetic

Q : Do there exist real numbers x & y s.t

$$x + 2y \leq 5$$
$$\& \quad x - 8y > 3$$
$$\& \quad \neg(x + 4y < 8)$$

$$\Longrightarrow$$

$$x + 2y \leq 5$$
$$x - 8y > 3$$
$$x + 4y \geq 8$$

Submit to favorite LP solver

$\exists x . \exists y .$ ______

$\forall x . \exists y . \forall z . \exists w$ ______

Eliminate w

innermost ~~variable~~

"Quantifier elimination"

Conjunction of linear equalities & inequalities

$x + 8y \leq 3$, $x - 2z = 9$, ...

$xx^2 - 4yz \geq 3$

$x = y$

$x \geq y$

$y \geq x$

Don't worry about equalities

Q $\exists y.\ \exists x.\ \ x + 2y \leq 5$

$x - 8y > 3$

$x + 4y \geq 8$

1 $x \leq 5 - 2y$

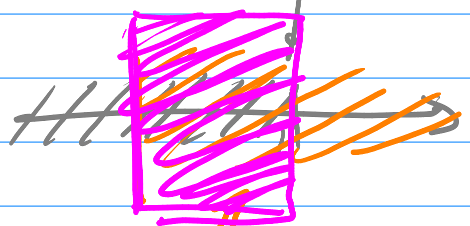
$x > 3 + 8y$

$x \geq 8 - 4y$

2 $x \leq 5 - 2y$

$8 - 4y \leq x$

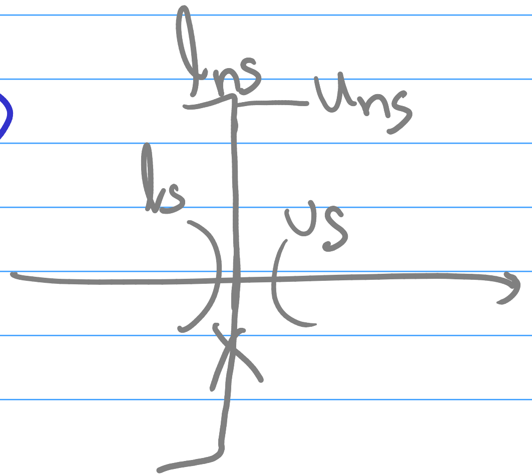
$3 + 8y < x$



$$\exists x .\quad \begin{array}{l} x < u_s \\ x \leq u_{ns} \\ l_{ns} \leq x \\ l_s < x \end{array} \iff \begin{array}{l} l_{ns} < u_s \\ l_{ns} \leq u_{ns} \\ l_s < u_s \\ l_s < u_{ns} \end{array}$$

Proof

$\Rightarrow$



$\Leftarrow$ If there is a gap, then you can pick x

$$\forall x \; \underline{\qquad\qquad} \iff \neg \cdot \underbrace{\exists x \cdot \neg \; \underline{\qquad\qquad}}$$

$$\forall x \cdot \; x < 3y \;\&\; y < 8x$$

$$\neg \cdot \exists x \cdot \; \neg(x < 3y \;\&\; y < 8x)$$

$$\neg \exists x \cdot (x \geq 3y \text{ or } y \geq 8x)$$

FO

Decidable / Quantifier elimination

$$\begin{tikzcd}
\text{LRA} \arrow[r] \arrow[d, dash] & \text{LIA} \arrow[d, dash] \\
\text{RA} & \text{IA}
\end{tikzcd}$$

undecidable

Matiyasevich's theorem

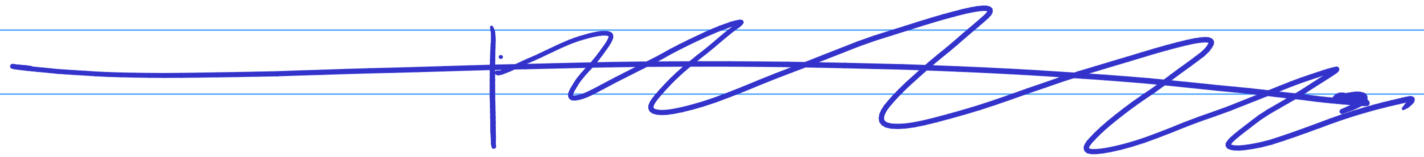
Hilbert's 10^{th} problem

$\forall x.\ \exists y.\ x^2 = y$: sat

$\underset{\mathbb{R}}{\forall x}.\ \exists \underset{\mathbb{R}}{y}.\ x = y^2$: unsat

First order $\forall x.\ \exists y.\ x < y$? Yes : $y = x+1$

Second order $\forall X.\ \exists y.\ \forall x \in X.\ x < y$? No



There exist exactly two values of x s.t. $\varphi(x)$

$$\exists x.\ \exists y.\ \varphi(x) \ \&\ \varphi(y) \ \&\ \forall z.\ \varphi(z) \Rightarrow (x=z \text{ or } y=z)$$

$$\&\ x \neq y$$

Ex:

$\exists x \in \mathbb{R} \quad 3 < x \quad \text{and} \quad x < 4$? Yes

$\exists x \in \mathbb{Z} \quad 3 < x \quad \text{and} \quad x < 4$? No

$\exists x . \exists y . z \quad x = y + z \quad \text{and} \quad 3 < x \quad \text{and} \quad y + z > 3$?

$\underline{\exists x.}$ $y = x + x + x$

$\varphi(y)$

$y \bmod 3 = 0$

$\boxed{\text{Theorem 6.12 from Sipser's textbook}}$