$u - 8 \leq 16 + w \iff u - w \leq 24$

Difference Logic

$u - 8 \leq v \qquad v \leq 16 + w$

Ex: $\exists u, v, w$ s.t. $u - v \leq 8 \quad v - w \leq 16 \quad u - w \geq 32$?

$\Rightarrow u - w \leq 24$

$u - w = 0 \iff u - w \leq 0$ and $u - w \geq 0$

Difference logic: ① All variables range over $\mathbb{Z}$ (or $\mathbb{R}$)

② Formula is a conjunction of constraints

③ Each constraint is of the form $u - v \leq c$,

for some variables u, v

constant c.

$\exists u, v, w$ s.t. $u - v \le 8 \quad v - w \le 16 \quad u - w \ge 32$?

$$-(u-w) \le -32$$

$$w - u \le -32$$

$$\begin{tikzcd} w \arrow[r, "16"] & v \arrow[r, "8"] & u \arrow[ll, bend right, "-32"'] \end{tikzcd}$$

1. Vertices = all variables
2. $u \xrightarrow{c} v$ edge whenever constraint $v - u \le c$

From this graph, we can recover the original formula.

$\therefore$ The graph itself encodes satisfiability "in some form".

Conjecture Difference logic formula φ is satisfiable

iff no negative wt. cycles in G_φ.

Ex 1:

$$\begin{tikzcd} u \arrow[r, bend right, "4"'] & v \arrow[l, bend right, "-8"'] \end{tikzcd}$$

$$\left.\begin{array}{l} v - u \le 4 \\ u - v \le -8 \end{array}\right] \Rightarrow \underline{\underline{0 \le -4}}$$

Ridiculous

Ex2:

$$\begin{tikzcd} u \arrow[r, bend right, "4"'] & v \arrow[l, bend right, "8"'] \end{tikzcd}$$

$$v - u \leq 4$$
$$u - v \leq 8$$

Satisfiable, set both to 0

Ex3:

$$\begin{tikzcd} u \arrow[r, bend right, "-4"'] & v \arrow[l, bend right, "-8"'] \end{tikzcd}$$

$$\begin{array}{r} v - u \leq -4 \\ u - v \leq -8 \\ \hline 0 \leq -12 \end{array}$$

Unsat

Exh :

$$u \xrightarrow{18} v \xrightarrow{27} w$$

$$\frac{v-u \le 18 \qquad w-v \le 27}{w-u \le 45}$$

Claim : If G_φ contains a path $u \overset{c}{\rightsquigarrow} v$

then $\varphi \Rightarrow v-u \le c$

Claim: If G_φ contains a negative weight cycle,

$$u \overset{c}{\rightsquigarrow} u \text{, with } c \lneq 0,$$

then $\varphi \Rightarrow u - u \le c$

$$\Rightarrow 0 \le c \quad \text{Preposterous}$$

& φ is unsat

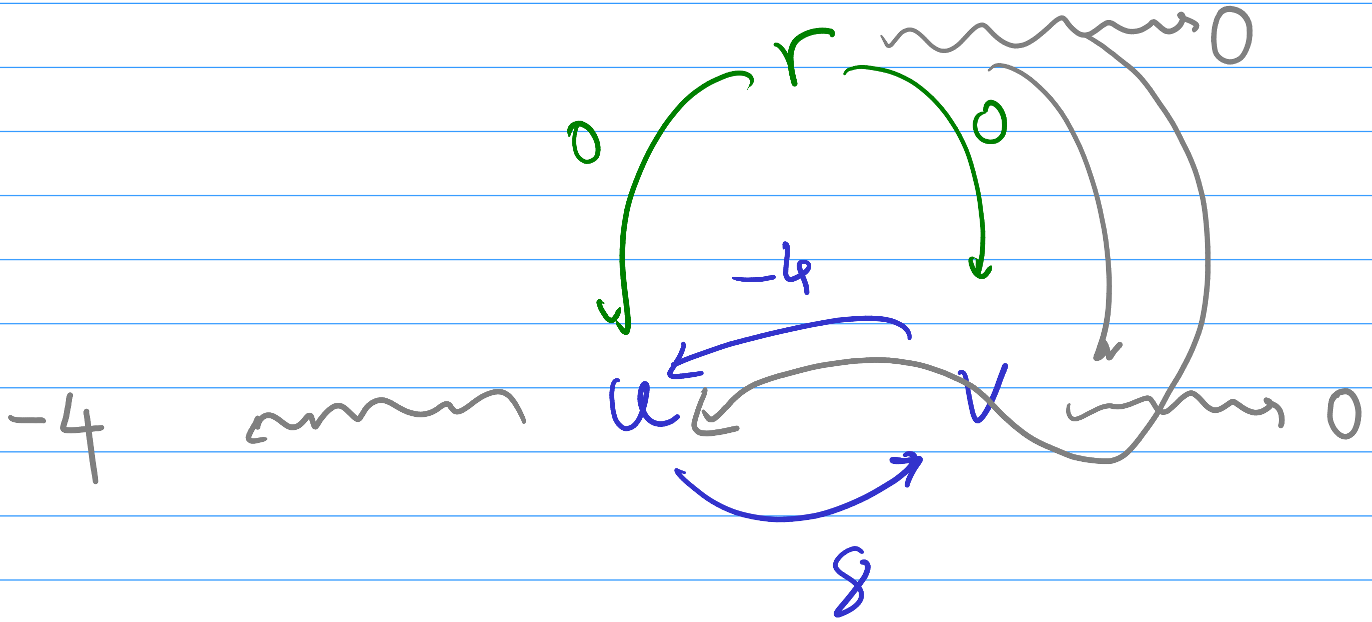
Ex 4 :

$$\begin{tikzcd} u \arrow[r, bend right, "8"'] & v \arrow[l, bend right, "-4"'] \end{tikzcd}$$

u		-4	-3	-2	$\dots$	0
v		$\underline{\underline{0}}$	1	2		4

$$v - u \le 8$$

$$u - v \le -4$$



Claim: Construct the assignment $m \{ u \mapsto \mathrm{dist}(r,u) \mid u \}$

$m \models \varphi$.

Theory of Equality

Unset

Ex: $\exists x \quad y \quad z \in \mathbb{Z} \cdot \quad x = y \quad x \neq z \quad y = z$?

$\exists x \quad y \quad z \in \mathbb{R} \cdot \quad x = y \quad x \neq z \quad y = z$?

$\exists x \; y \; z$ strings

$\exists x \; y \; z$ objects

Theory of equality

$\exists v_1\ v_2 \cdots\ u_1\ u_2 \cdots \in$ set of objects of your choosing

$\varphi = \quad v_1 = v_2$ and $v_3 = v_4$ and $\cdots$ and $v_k = v_{k+1}$

and $u_1 \neq u_2$ and $u_3 \neq u_4$ and $\cdots$ and $u_m \neq u_{m+1}$

?

$\varphi : \exists x\ y\ z \in \mathbb{D} \cdot x = y \quad x \neq z \quad y = z\ ?$

$x \text{ --- } y \text{ --- } z$

$G_\varphi = \{ u \text{ --- } v \mid \text{whenever } (u = v) \in \varphi \}$

<u>Claim</u>: if $u \neq v \in \varphi$ & u & v are in the same SCC in G_φ, then φ is unsat.

Equality with Uninterpreted Functions

Ex

$\exists f : \mathbb{Z} \to \mathbb{Z} \; \exists x \in \mathbb{Z} \; . \quad f(x) = x \quad \text{and} \quad f(f(x)) \neq f(x)?$

$f(x) \neq f(x)$

$f(x) \neq x$

$\exists f : \mathbb{R} \to \mathbb{R} .\ \exists x \in \mathbb{R} .\quad f(x) = x \text{ and } f(f(x)) \neq f(x)?$

"uninterpreted"

unsat

$\exists f : \text{string} \to \text{string}\ \exists x \in \text{string} .\quad f(x) = x \text{ and } f(f(x)) \neq f(x)?$

float $x_0 = \text{input}$

$x_1 = \text{sqrt}(x_0)$

$x_2 = \text{sqrt}(x_1)$

if $(x_1 = x_0$ & $x_2 \neq x_1)$ then crash!