

DPLL Algorithm

Ex:

$$x_1 \vee x_4 \quad (1)$$

$$\text{and } x_1 \vee \bar{x}_3 \vee \bar{x}_8 \quad (2)$$

$$\text{and } x_1 \vee x_8 \vee x_{12} \quad (3)$$

$$\text{and } x_2 \vee x_{11} \quad (4)$$

$$\text{and } \bar{x}_7 \vee \bar{x}_3 \vee x_9 \quad (5)$$

$$\text{and } \bar{x}_7 \vee x_8 \vee \bar{x}_9 \quad (6)$$

$$\text{and } x_7 \vee x_8 \vee \bar{x}_{10} \quad (7)$$

$$\text{and } x_7 \vee x_{10} \vee \bar{x}_{12} \quad (8)$$

choice #1: $\overline{x_1}$
↓
 x_4

DPLL Algorithm

Ex:

~~x_1~~ \vee ~~x_4~~ \vee ~~x_5~~

and ~~x_1~~ \vee $\overline{x_3}$ \vee $\overline{x_8}$ (2)

and ~~x_1~~ \vee x_8 \vee x_{12} (3)

and x_2 \vee x_{11} (4)

and $\overline{x_7}$ \vee $\overline{x_3}$ \vee x_9 (5)

and $\overline{x_7}$ \vee x_8 \vee $\overline{x_9}$ (6)

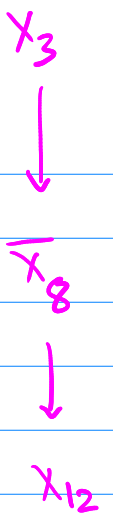
and x_7 \vee x_8 \vee $\overline{x_{10}}$ (7)

and x_7 \vee x_{10} \vee $\overline{x_{12}}$ (8)

$$\overline{x_1} \xrightarrow{\textcircled{1}} x_5$$

$$x_3 \xrightarrow{\textcircled{2}} \overline{x_8}$$

Choice #12 :



DPLL Algorithm

Ex:

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee ~~x_6~~

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee ~~x_6~~ \vee ~~x_7~~

and $x_2 \vee x_{11}$ (4)

and $\bar{x}_7 \vee \bar{x}_8 \vee x_9$ (5)

and $\bar{x}_7 \vee \bar{x}_8 \vee \bar{x}_9$ (6)

and $x_7 \vee \bar{x}_8 \vee \bar{x}_{10}$ (7)

and $x_7 \vee x_{10} \vee \bar{x}_{11}$ (8)

Choice #3:

\bar{x}_2
↓
 x_{11}

DPLL Algorithm

Ex:

and ~~x_1~~ \vee ~~x_2~~ \vee x_3 \vee x_4

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee x_5 \vee x_6

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee x_6

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee x_6

and \bar{x}_7 \vee ~~x_8~~ \vee x_9 (5)

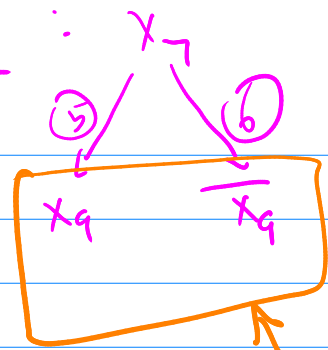
and \bar{x}_7 \vee ~~x_8~~ \vee \bar{x}_9 (6)

and x_7 \vee ~~x_8~~ \vee \bar{x}_{10} (7)

and x_7 \vee x_{10} \vee ~~x_8~~ (8)

DPLL Algorithm

Choice #4



Ex:

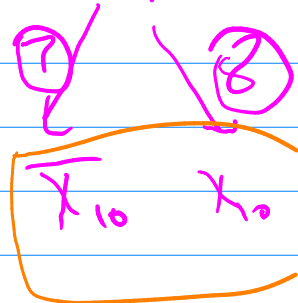


(CONFLICT!)

Choosing x_7 was bad. so $\overline{x_7}$

DPLL Algorithm

Ex:



CONFLICT!

DPLL Algorithm

Choosing \bar{x}_2 led
to issues. So,
let's choose x_2 .

Ex:

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee ~~x_6~~

and ~~x_1~~ \vee ~~x_2~~ \vee ~~x_3~~ \vee ~~x_4~~ \vee ~~x_5~~ \vee ~~x_6~~ \vee ~~x_7~~

and ~~x_2~~ \vee ~~x_{11}~~ \vee ~~x_{12}~~

and \bar{x}_7 \vee ~~x_8~~ \vee x_9 \vee ~~x_{10}~~ \vee ~~x_{11}~~ \vee ~~x_{12}~~ (5)

and \bar{x}_7 \vee ~~x_8~~ \vee \bar{x}_9 \vee ~~x_{10}~~ \vee ~~x_{11}~~ \vee ~~x_{12}~~ (6)

and x_7 \vee ~~x_8~~ \vee \bar{x}_{10} \vee ~~x_{11}~~ \vee ~~x_{12}~~ (7)

and x_7 \vee x_{10} \vee ~~x_{11}~~ \vee ~~x_{12}~~ (8)

DPLL(φ) :

- While there is a unit clause

$c = l$,

$\varphi := \varphi[l \mapsto \text{true}]$

- If you reach an empty clause
then unsat

- Pick variable x . Pick polarity $v \in \{\text{true}, \text{false}\}$

$\text{DPLL}(\varphi[x \mapsto v])$ or

$\text{DPLL}(\varphi[x \mapsto \bar{v}])$

Karnaugh maps

Quine

1952

~ 10 vars

DP/DPLL

1960/1962

~ 10 vars

CHAFF

Watched
literals

VSIDS

Restart

~ 10 k vars

GRASP

Marques Silva

Khrem Sakallah

CDCL

1 k vars

Bryant

ROBDDs

1980s

100 vars

Polynomial Time Algorithms for 2-SAT

Ex: $(a \vee b)$ and $(\bar{a} \vee b)$ and $(\bar{a} \vee \bar{b})$ and $(a \vee \bar{c})$